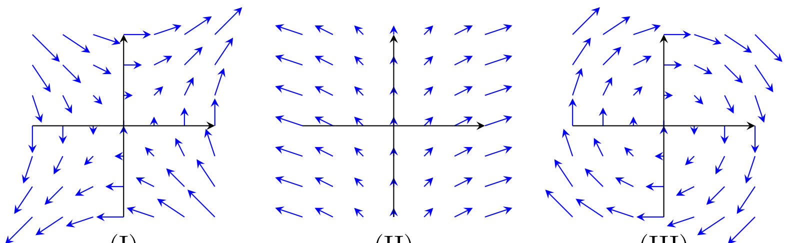
Math 241 Exam 4 Fall 2019 **Justin Wyss-Gallifent**

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.
Important: Out of all the integrals that ask to be evaluated only one of them actually requires integration, the rest use theorems which make them fast.

Please put problem 1 on answer sheet 1

1. Consider the following three pictures of vector fields. Each axis goes from -3 to 3 in each of the x and y directions and the vectors are anchored at integer coordinates. [20 pts]



(I) (II) (III)

For each of the following (a), (b), and (c), plug in a single point (x, y) which allows you to identify which of the vector fields (I), (II), or (III) matches the vector field.

(a) $\mathbf{F}(x, y) = 0.3y\,\mathbf{i} - 0.3x\,\mathbf{j}$
(b) $\mathbf{F}(x, y) = 0.3y\,\mathbf{i} + 0.3x\,\mathbf{j}$
(c) $\mathbf{F}(x, y) = 0.3x\,\mathbf{i} + 0.3\,\mathbf{j}$

Extra Credit: At the bottom of the first sheet put the date, time, building (code or full name) and room number of your final exam. [≤3 pts]

Please put problem 2 on answer sheet 2

2. Parts (a) and (b) are unrelated.

(a) Let Σ be the sphere with equation $x^2 + y^2 + z^2 = 9$ with inwards orientation. Evaluate: [8 pts]

$$\iint_\Sigma (x\,\mathbf{i} + 2y\,\mathbf{j} + 3z\,\mathbf{k}) \cdot \mathbf{n}\,dS$$

(b) Suppose C is the curve parametrized by $\mathbf{r}(t) = t^2\,\mathbf{i} + t\,\mathbf{j} + 2\,\mathbf{k}$ for $0 \le t \le 2$. Evaluate: [12 pts]

$$\int_C y\,dx + z\,dy + z\,dz$$

Please put problem 3 on answer sheet 3

3. Suppose Σ is the portion of the parabolic sheet $y = 4 - x^2$ in the first octant and below $z = 3$. The mass density at each point is given by $f(x, y, z) = x + y + z$. Set up an iterated integral for the mass of Σ. [20 pts]
Do Not Evaluate This Integral.

TURN OVER FOR PROBLEMS 4 AND 5

Please put problem 4 on answer sheet 4

4. Parts (a) and (b) are unrelated.

   (a) Let C be the clockwise triangle with vertices $(5,0)$, $(5,10)$, and $(0,10)$. Evaluate: [10 pts]

$$\int_C (x+2y)\,dx + (4x+y)\,dy$$

   (b) Let C be the straight-line segment from $(1,2,-2)$ to $(8,0,5)$. Evaluate: [10 pts]

$$\int_C (y\,\mathbf{i} + x\,\mathbf{j} + 1\,\mathbf{k}) \cdot d\mathbf{r}$$

Please put problem 5 on answer sheet 5

5. Suppose C is the intersection of the plane $2x+y=10$ with the cylinder $x^2+z^2=9$ with counterclockwise orientation when viewed towards the origin from the positive y-axis. Apply Stokes' Theorem to the integral [20 pts]

$$\int_C (x+y)\,dx + y^2\,dy + (xyz)\,dz$$

   and proceed until you have an iterated double integral.
   Do Not Evaluate This Integral.

The End of the Exam and the TA Section List

Stephen S.	0311 $\Leftrightarrow$ 10:00	0321 $\Leftrightarrow$ 11:00
Ke	0312 $\Leftrightarrow$ 10:00	0322 $\Leftrightarrow$ 11:00
S. Gilles	0331 $\Leftrightarrow$ 12:00	0341 $\Leftrightarrow$ 1:00
Chenzhi	0332 $\Leftrightarrow$ 12:00	0342 $\Leftrightarrow$ 1:00