Your Name NEATLY:

MATH241 Exam 4 Fall 2021 (Justin Wyss-Gallifent)

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any Calc 3 related work, words or ideas which are relevant to the problem.

1. Extra Credit: Write down your TA and the date, time, building code and room number of your final exam. [2 or 0 pts]

   Solution:

2. Suppose C is a piece of wire in the shape of a parabola with equation $y = 2x^2$ for $0 \leq x \leq 2$. Here x, y are in centimeters. If the electrical charge density at (x, y) is given by $f(x, y) = 4x$ coulombs per centimeter calculate the total electrical charge on the wire. [15 pts]

   Evaluate

   Solution:

3. Suppose an object follows the clockwise triangular path from $(0,0)$ to $(3,1)$ to $(1,0)$ and back to $(0,0)$. Use Green's Theorem to write down an iterated double integral for the work done on the object by the vector field: [15 pts]

$$\vec{F}(x,y) = 0.5y^2\hat{\boldsymbol{\imath}} - xy\hat{\boldsymbol{\jmath}}$$

Do Not Evaluate

Solution:

4. Suppose Σ is the portion of $y = 4 - x^2$ in the first octant and below $z = 5$. Write down an iterated double integral for the surface area of Σ. [15 pts]

Do Not Evaluate

Solution:

5. Let C be any curve from $(1, 2, 3)$ to $(4, 3, -2)$. Evaluate and simplify: [10 pts]

$$\int_C \left(\frac{2x}{y} + ze^{xz}\right) dx + \left(-\frac{x^2}{y^2}\right) dy + xe^{xz}\, dz$$

Evaluate

Solution:

6. Suppose Σ is the part of the cone $y = 3 - \sqrt{x^2 + z^2}$ with $y \geq 0$ along with the base $x^2 + z^2 \leq 9$ on the xz-plane. Suppose Σ has inwards orientation. [10 pts]

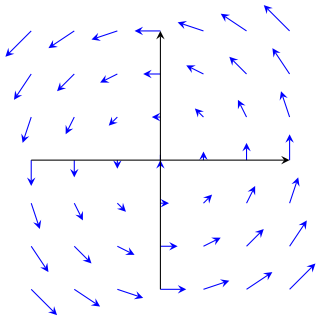
   Evaluate the integral:

$$\iint_\Sigma (4x\hat{\boldsymbol{\imath}} + 3y\hat{\boldsymbol{\jmath}} - z\hat{\boldsymbol{k}}) \cdot \vec{n}\, dS$$

Evaluate

Solution:

7. Explain briefly why the following vector field is not conservative: [5 pts]



Solution:

8. Suppose we have $f(x, y, z) = x^2y - xz$. Only one of the following makes sense. Circle the one that does and then calculate it. You do not need to justify how you made your choice. [10 pts]

$$\nabla \cdot (\nabla f) \qquad \text{and} \qquad \nabla(\nabla \cdot f)$$

Solution:

9. Suppose C is the intersection curve of the paraboloid $z = x^2 + y^2$ with the cylinder $r = \cos\theta$ with clockwise orientation when viewed from above. Apply Stokes' Theorem to the integral [20 pts]

$$\int_C (x + z)\, dx + x^2\, dy + xy\, dz$$

and proceed until you have an iterated double integral.

Do Not Evaluate

Solution: