MATH241 Fall 2023 Exam 4 (Justin W-G)

Name (Neatly):	
UID (Neatly):	

Instructions:

1. Please do all problems on the pages and in the spaces provided. This exam will be scanned into Gradescope and if your answers are not in the correct locations they will not be found or graded!
2. Only simplify Calculus 3 related calculations unless otherwise specified.

Extra Credit: Write the date, time, building and room of your final exam in the table below. If taking with ADS, write "ADS" in the Building and Room spaces. [$\leq$ 2 pts]

Solution:

Date	Time	Building	Room

1. Write TRUE or FALSE in the box to the right. No justification is required. Unreadable or ambiguous answers will be marked as incorrect. [10 pts]

Solution:

Statement	TRUE/FALSE
You absolutely must use FTOLI for integrating conservative vector fields.	
The surface integral of a vector field requires an oriented surface.	
Green's Theorem applies in 3D space.	
$\nabla \bar{\boldsymbol{F}}$ makes sense.	
Surface area is the surface integral of $1\,\hat{\boldsymbol{\imath}}$.	

2. Let C be the curve parametrized by $\bar{\boldsymbol{r}}(t) = 3\cos t\,\hat{\boldsymbol{\imath}} + 3\sin t\,\hat{\boldsymbol{\jmath}}$ for $0 \le t \le \frac{2\pi}{3}$. Without integrating, what is the value of $\int_C 7\,ds$ and why? [5 pts]

Solution:

3. Let Σ be the disk $x^2 + z^2 = 9$ in the plane $y = 42$, oriented in the positive y direction. Without integrating, what is the value of $\iint_\Sigma (10x\,\hat{\boldsymbol{\imath}} + 0\,\hat{\boldsymbol{\jmath}} + xy^2 z\,\hat{\boldsymbol{k}}) \cdot n\,dS$ and why? [5 pts]

Solution:

4. Let Σ be the part of the cylinder $y^2 + z^2 = 9$ between $x = 1$ and $x = 3$. If the mass density at a point is given by $f(x, y, z) = x$, write down and evaluate the integral for the mass of Σ. [20 pts]

$\boxed{\textbf{You Should Evaluate Your Result!}}$

Solution:

5. Suppose $\bar{\boldsymbol{F}}(x, y, z) = 1\,\hat{\boldsymbol{\imath}} + z\,\hat{\boldsymbol{\jmath}} + y\,\hat{\boldsymbol{k}}$ and C is the straight line segment from $(1, 2, 3)$ to $(8, 10, 100)$. [5 pts]
Evaluate the line integral:

$$\int_C \bar{\boldsymbol{F}} \cdot d\bar{\boldsymbol{r}}$$

$\boxed{\textbf{You Should Evaluate Your Result!}}$

Solution:

6. Use Green's Theorem to evaluate the line integral $\int_C 4y\,dx + 6x\,dy$ where C is the curve following $y = x^2$ from $(0, 0)$ to $(2, 4)$, then following a straight line to $(2, 0)$ then following a straight line back to $(0, 0)$. [15 pts]

$\boxed{\textbf{You Should Evaluate Your Result!}}$

Solution:

7. Let Σ be the part of $z = 2 - \sqrt{x^2 + y^2}$ above the xy-plane as well as the disk $x^2 + y^2 \leq 4$ in the xy-plane, oriented inwards. Apply the Divergence Theorem to construct an iterated triple integral in cylindrical coordinates for the following: [15 pts]

$$\iint_\Sigma (xy\,\hat{\boldsymbol{\imath}} + y\,\hat{\boldsymbol{\jmath}} + xz\,\hat{\boldsymbol{k}}) \cdot \bar{\boldsymbol{n}}\,dS$$

$\boxed{\textbf{You Should Not Evaluate Your Result!}}$

Solution:

8. Let Σ be the part of the vertical plane $x + 2y = 4$ in the first octant and below $z = 5$. Let C be the edge of Σ with clockwise orientation when viewed from the first octant looking towards the origin. Consider the integral: [25 pts]

$$\int_C z^2\,dx + x\,dy + z\,dz$$

Use Stokes' Theorem to rewrite the line integral as a surface integral and then as an iterated double integral.

$\boxed{\textbf{You Should Not Evaluate Your Result!}}$

Solution: