Math 241 Exam 4 Spring 2017

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Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) Evaluate \( \int_C 8y \, ds \) where \( C \) is parametrized by \( r(t) = t^2 \, \mathbf{i} + t \, \mathbf{j} \) for \( 0 \leq t \leq 3 \). [10 pts]

(b) Let \( C \) be parametrized by \( r(t) = t^2 \, \mathbf{i} + (t^3 + 1) \, \mathbf{j} + (2t - 1) \, \mathbf{k} \) for \( 1 \leq t \leq 3 \). Evaluate \[ \int_C (2xy \, \mathbf{i} + (x^2 + z) \, \mathbf{j} + (y + 1) \, \mathbf{k}) \cdot d\mathbf{r} \] [10 pts]

Extra Credit: At the bottom of the first sheet put the date, time, building (code or full name) and room number of your final exam. [+3 pts]

Please put problem 2 on answer sheet 2

2. Set up the integral for the mass of \( \Sigma \), the part of the cylinder \( x^2 + z^2 = 4 \) between \( y = 0 \) and \( y = 3 \) and in the first octant the mass density at \( (x, y, z) \) is given by \( \delta(x, y, z) = x^2 \). Proceed until you have an iterated double integral but do not evaluate. [20 pts]

Please put problem 3 on answer sheet 3

3. Let \( C \) be the clockwise triangle with corners \((0, 0), (3, 3)\) and \((3, 6)\). Use Green’s Theorem to evaluate the integral \[ \int_C x^3 \, dx + x^2 \, dy \] [20 pts]

Please put problem 4 on answer sheet 4

4. Let \( \Sigma \) be the part of the parabolic sheet \( z = 9 - x^2 \) above the rectangle with corners \((0, 0, 0), (2, 0, 0), (0, 5, 0), (2, 5, 0)\). Let \( C \) be the edge of \( \Sigma \) with clockwise orientation when viewed from above. Apply Stokes’ Theorem to the line integral \[ \int_C (xz \, \mathbf{i} + x^2 \, \mathbf{j} + xz \, \mathbf{k}) \cdot d\mathbf{r} \] Parametrize the resulting surface and proceed until you have an iterated double integral but do not evaluate. [25 pts]

Please put problem 5 on answer sheet 5

5. Let \( \Sigma \) be the hollow tetrahedron (four-sided figure) with corners \((0, 0, 0), (1, 0, 0), (0, 1, 0)\) and \((0, 0, 1)\) with outwards orientation. Use the Divergence Theorem to rewrite \[ \iint_\Sigma (x^2 \, \mathbf{i} + xz \, \mathbf{j} + 5x \, \mathbf{k}) \cdot \mathbf{n} \, dS \] as a triple integral and write as an iterated integral but do not evaluate. [15 pts]

The End and the TA Section List

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