Math 241 Exam 4 Spring 2019 **Justin Wyss-Gallifent**

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. Parts (a) and (b) are unrelated.

   (a) Is $\mathbf{F}(x, y, z) = xy\,\mathbf{i} + y\,\mathbf{j} - 2xz\,\mathbf{k}$ conservative? Justify your answer. [10 pts]

   (b) Given $f(x, y, z) = x^2 z + \frac{x}{y}$ only one of the following makes sense. Calculate the one that does: [10 pts]

   $$\nabla(\nabla f) \qquad \nabla \cdot (\nabla f) \qquad \nabla \cdot (\nabla \times f)$$

   Extra Credit: At the bottom of the first sheet put the date, time, building (code or full name) and room number of your final exam. [+3 pts]

Please put problem 2 on answer sheet 2

2. Let C be the curve with parametrization given by:

   $$\mathbf{r}(t) = t^2\,\mathbf{i} + (1 - t)\,\mathbf{j} \text{ for } 1 \leq t \leq 3.$$

   The Fundamental Theorem of Line Integrals can be used for exactly one of the following. Use it for that one and then do the other one another way.

   (a) Evaluate $\int_C x\,dx + x\,dy$. [10 pts]

   (b) Evaluate $\int_C y\,dx + x\,dy$. [10 pts]

Please put problem 3 on answer sheet 3

3. Parts (a) and (b) are unrelated.

   (a) Use the Divergence Theorem to evaluate the following integral where Σ is a cube with side length 3, oriented outwards: [8 pts]

   $$\iint_\Sigma (2x\,\mathbf{i} + x\,\mathbf{j} + 10z\,\mathbf{k}) \cdot \mathbf{n}\,dS$$

   (b) Evaluate $\int_C x\,dx + x^2\,dy$ where C is triangle with corners $(0, 0)$, $(0, 4)$ and $(2, 4)$, oriented clockwise. [12 pts]

Please put problem 4 on answer sheet 4

4. Let Σ be the portion of the plane $x + z = 4$ in the first octant between $y = 3$ and $y = 5$. If the mass density at a point is $f(x, y, z) = yz$ write down an iterated integral corresponding to the mass of Σ. **Do Not Evaluate This Integral.** [20 pts]

Please put problem 5 on answer sheet 5

5. Let C be the intersection of the cylinder $x^2 + z^2 = 4$ with the plane $y + z = 4$ with counterclockwise orientation when viewed from the positive y-axis. Apply Stokes' Theorem to the integral $\int_C xz\,dx + x\,dy + y^2\,dz$ and proceed until you have an iterated double integral. [20 pts]
   Do Not Evaluate This Integral.

The End and the TA Section List

Chenzhi	0311 ↔ 8:00	0321 ↔ 9:30
Corry	0312 ↔ 8:00	0322 ↔ 9:30
Noah	0331 ↔ 11:00	0341 ↔ 12:30
Papia	0332 ↔ 11:00	0342 ↔ 12:30