

## MATH241 Spring 2023 Exam 4 (Justin W-G)

Name (Neatly)	
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**Instructions:**

1. Please do all problems on the pages and in the spaces provided. This exam will be scanned into Gradescope and if your answers are not in the correct locations they will not be found or graded!
2. Only simplify Calculus 3 related calculations.

Extra Credit: Write the date, time, building and room of your final exam in the table below. If taking with ADS, write "ADS" in the Building and Room spaces. [ $\leq 2$  pts]

**Solution:**

Date	Time	Building	Room

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**ADS - CRITICAL - PLEASE SCAN THIS PAGE!**

1. Write T for True or F for False in the box to the right. No justification is required. Unreadable or ambiguous letters will be marked as incorrect. [10 pts]

**Solution:**

Statement	T or F
$\iint_{\Sigma} 1 \, dS$ yields surface area.	
The line integral of a conservative vector always equals zero.	
Stokes' Theorem applies to integrals of the form $\iiint_D f(x, y, z) \, dV$ .	
The Fundamental Theorem of Line Integrals requires a conservative vector field.	
The Divergence Theorem got its name because the answer always diverges to $\infty$ .	

2. Let  $\Sigma$  be the part of  $x + y = 2$  in the first octant with  $0 \leq z \leq 5$ . The following integral can be evaluated without any integration. Briefly explain and give the answer. [5 pts]

$$\iint_{\Sigma} 4 \, dS$$

**NOTE:** Zero points will be given for actually integrating!

**Solution:**

3. Prove that the following vector field is not conservative: [5 pts]

$$\vec{F}(x, y, z) = xz \hat{i} + (z - y) \hat{j} + y \hat{k}$$

**Solution:**

4. The following integrals can be calculated using a theorem followed by an easy calculation. For each, state the theorem and evaluate.

(a) Let  $C$  be parameterized by  $\vec{r}(t) = t^2 \hat{i} + t \cos t \hat{j}$  for  $0 \leq t \leq \pi$ . [6 pts]

$$\int_C (1 + y) dx + x dy$$

**Solution:**

(b) Let  $C$  be the clockwise triangle with corners  $(0, 0)$ ,  $(2, 2)$ , and  $(6, 0)$ . [6 pts]

$$\int_C 2xy dx + (x^2 - 7x) dy$$

**Solution:**

(c) Let  $\Sigma$  be the part of the cone  $z = 2 - \sqrt{x^2 + y^2}$  above the  $xy$ -plane along with the disk  $x^2 + y^2 \leq 4$  in the  $xy$  plane. Assume  $\Sigma$  is oriented outwards. [8 pts]

$$\iint_{\Sigma} (5x \hat{i} + 7x \hat{j} - 10z \hat{k}) \cdot \vec{n} dS$$

**Solution:**

5. Let  $C$  be the line segment from  $(0, 1)$  to  $(8, 3)$ . Evaluate the line integral:

[10 pts]

$$\int_C (x \hat{i} + xy \hat{j}) \cdot d\vec{r}$$

**YOU MUST EVALUATE THIS INTEGRAL!**

**Solution:**

6. Let  $\Sigma$  be the part of the parabolic sheet  $y = x^2$  between  $x = -1$  and  $x = 2$  and with  $0 \leq z \leq 7$ . [15 pts]  
Write down an iterated double integral for the surface area of  $\Sigma$ .

**DO NOT EVALUATE!**

**Solution:**

7. Let  $\Sigma$  be the part of the plane  $2x + 3y + z = 6$  in the first octant and oriented upwards. Write [15 pts]  
down an iterated double integral for the rate of flow of  $\vec{F}(x, y, z) = 0\hat{i} + 0\hat{j} + z\hat{k}$  through  $\Sigma$ .

**DO NOT EVALUATE!**

**Solution:**

8. Let  $C$  be the intersection of  $x^2 + z^2 = 9$  with  $x + y = 5$  and with counterclockwise orientation [20 pts] when viewed from the positive  $y$ -axis. Use Stokes' Theorem to obtain an iterated double integral which calculates:

$$\int_C x \, dx + xz \, dy + xy \, dz$$

**DO NOT EVALUATE!**

**Solution:**