1. In each of the following problems it’s possible to figure out if the surface integral $\iint_{\Sigma} f \, dS$ is positive, negative or zero without doing any work. For each, figure this out and explain. Hint: What does this surface integral measure?

(a) $\Sigma$ is the part of the paraboloid $z = 4 - x^2 - y^2$ above the $xy$-plane and $f(x, y, z) = z$.
(b) $\Sigma$ is the part of the cylinder $x^2 + z^2 = 9$ between $y = 0$ and $y = 3$ and $f(x, y, z) = x$.
(c) $\Sigma$ is the sphere $x^2 + y^2 + (z - 1)^2 = 1$ and $f(x, y, z) = z - 2$.

2. In each of the following problems it’s possible to figure out if the surface integral $\iint_{\Sigma} \mathbf{F} \cdot \mathbf{n} \, dS$ is positive, negative or zero without doing any work. For each, figure this out and explain. Hint: What does this surface integral measure?

(a) $\Sigma$ is the part of the plane $z = 3$ with $-3 \leq x \leq 3$ and $-3 \leq y \leq 3$ and $\mathbf{F}(x, y, z) = y^2 \mathbf{k}$.
(b) $\Sigma$ is the part of the plane $z = 3$ with $-3 \leq x \leq 3$ and $-3 \leq y \leq 3$ and $\mathbf{F}(x, y, z) = y^2 \mathbf{j}$.
(c) $\Sigma$ is the sphere $x^2 + y^2 + z^2 = 9$ and $\mathbf{F}(x, y, z) = e^z \mathbf{k}$.

3. When doing $\iint_{\Sigma} \mathbf{F} \cdot \mathbf{n} \, dS$ we examine whether the vectors $\mathbf{r}_u \times \mathbf{r}_v$ agree with or disagree with the orientation of $\Sigma$. It is guaranteed that one of these things will be true. Suppose $\Sigma$ is the part of the plane $z = 2 - y$ in the first octant and between $x = 0$ and $x = 3$, oriented “upwards”. Suppose you parametrize using $x$ and $y$ and you find that $\mathbf{r}_x \times \mathbf{r}_y = 1 \mathbf{j} - 1 \mathbf{k}$. How do you know that you made a mistake?

4. There are often many ways to parametrize a surface and consequently many ways to solve surface integrals by parametrization. The half-cylinder $x^2 + z^2 = 9$ with $z \geq 0$ and $0 \leq y \leq 10$ can be parametrized either by $\mathbf{r}(\theta, y)$ or $\mathbf{r}(x, y)$. Rewrite the surface integral

$$\iint_{\Sigma} (y \mathbf{i} + x \mathbf{j} - xz \mathbf{k}) \cdot \mathbf{n} \, dS$$

as an iterated double integral twice, once using each of these parametrizations. Does one turn out to be nicer than the other? Which one and why?