**Math 241 Parametrization of Surfaces - Solutions**

1. (a) **Example:** $\Sigma$ is the part of the cone $z = \sqrt{x^2 + y^2}$ above the rectangle in the $xy$-plane with opposite corners $(1, 0)$ and $(2, 5)$.
   **Solution:** $\bar{r}(x, y) = x\,\hat{\imath} + y\,\hat{\jmath} + \sqrt{x^2 + y^2}\,\hat{k}$ with $1 \le x \le 2$ and $0 \le y \le 5$.

   (b) **Example:** $\Sigma$ is the part of the paraboloid $z = 9 - x^2 - y^2$ above the triangle in the $xy$-plane with corners $(0, 0)$, $(4, 0)$ and $(0, 2)$.
   **Solution:** $\bar{r}(x, y) = x\,\hat{\imath} + y\,\hat{\jmath} + (9 - x^2 - y^2)\,\hat{k}$ with $0 \le x \le 4$ and $0 \le y \le 2 - \frac{1}{2}x$.

   (c) **Example:** $\Sigma$ is the part of the plane $z = 20 - x - 2y$ above $R$, where $R$ is the region in the $xy$-plane between $y = x^2$ and $y = 4$.
   **Solution:** $\bar{r}(x, y) = x\,\hat{\imath} + y\,\hat{\jmath} + (20 - x - 2y)\,\hat{k}$ with $-2 \le x \le 2$ and $x^2 \le y \le 4$.

2. (a) **Example:** $\Sigma$ is the part of the cone $z = 2 + \sqrt{x^2 + y^2}$ inside the cylinder $x^2 + y^2 = 4$.
   **Solution:** $\bar{r}(r, \theta) = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath} + (2 + r)\,\hat{k}$ with $0 \le \theta \le 2\pi$ and $0 \le r \le 2$.

   (b) **Example:** $\Sigma$ is the part of the parabolic sheet $z = y^2$ inside the cylinder $r = \sin\theta$.
   **Solution:** $\bar{r}(r, \theta) = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath} + r^2\sin^2\theta\,\hat{k}$ for $0 \le \theta \le \pi$ and $0 \le r \le \sin\theta$.

   (c) **Example:** $\Sigma$ is the part of the plane $z = 20 - x - 2y$ in the first octant and inside $r = 2$.
   **Solution:** $\bar{r}(r, \theta) = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath} + (20 - r\cos\theta - 2r\sin\theta)\,\hat{k}$ with $0 \le \theta \le \frac{\pi}{2}$ and $0 \le r \le 2$.

3. (a) **Example:** $\Sigma$ is the part of the paraboloid $y = x^2 + z^2$ to the right of the square in the $xz$-plane with corners $(0, 0)$, $(2, 0)$, $(0, 2)$ and $(2, 2)$.
   **Solution:** $\bar{r}(x, z) = x\,\hat{\imath} + (x^2 + z^2)\,\hat{\jmath} + z\,\hat{k}$ with $0 \le x \le 2$ and $0 \le z \le 2$.

   (b) **Example:** $\Sigma$ is the part of the parabolic sheet $x = 16 - z^2$ inside the cylinder $y^2 + z^2 = 9$.
   **Solution:** $\bar{r}(r, \theta) = (16 - r^2\sin^2\theta)\,\hat{\imath} + r\cos\theta\,\hat{\jmath} + r\sin\theta\,\hat{k}$ with $0 \le \theta \le 2\pi$ and $0 \le r \le 3$.

4. (a) **Example:** $\Sigma$ is the part of the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 2$.
   **Solution:** $\bar{r}(z, \theta) = 3\cos\theta\,\hat{\imath} + 3\sin\theta\,\hat{\jmath} + z\,\hat{k}$ with $0 \le \theta \le 2\pi$ and $0 \le z \le 2$.

   (b) **Example:** $\Sigma$ is the part of the cylinder $x^2 + z^2 = 9$ between $y = 0$ and $y = 2$.
   **Solution:** $\bar{r}(y, \theta) = 3\cos\theta\,\hat{\imath} + y\,\hat{\jmath} + 3\sin\theta\,\hat{k}$ with $0 \le \theta \le 2\pi$ and $0 \le y \le 2$.

   (c) **Example:** $\Sigma$ is the part of the sphere $x^2 + y^2 + z^2 = 9$ below the cone $z = \sqrt{x^2 + y^2}$.
   **Solution:** $\bar{r}(\phi, \theta) = 3\sin\phi\cos\theta\,\hat{\imath} + 3\sin\phi\sin\theta\,\hat{\jmath} + 3\cos\phi\,\hat{k}$ with $0 \le \theta \le 2\pi$ and $\pi/4 \le \phi \le \pi$.

   (d) **Example:** $\Sigma$ is the part of the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 2$ and in the first octant.
   **Solution:** $\bar{r}(z, \theta) = 3\cos\theta\,\hat{\imath} + 3\sin\theta\,\hat{\jmath} + z\,\hat{k}$ for $0 \le \theta \le \pi/2$ and $0 \le z \le 2$.

   (e) **Example:** $\Sigma$ is the part of the sphere $x^2 + y^2 + z^2 = 9$ above the $xy$-plane.
   **Solution:** $\bar{r}(\phi, \theta) = 3\sin\phi\cos\theta\,\hat{\imath} + 3\sin\phi\sin\theta\,\hat{\jmath} + 3\cos\phi\,\hat{k}$ with $0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi/2$.