Math 241 Section 11.1: 3D, Points, Axes, Spheres, Distance
Dr. Justin O. Wyss-Gallifent

1. Goal/Intro: Most of MATH 241 (Multivariable Calculus) takes place in 3D space so we need to understand visually how all this works.

2. In addition to the x and y axis we add an extra axis, the z-axis. We rearrange so that the z-axis is pointing up. The reason for this is that most of our functions are of the form $z = f(x, y)$ and we're used to the dependent variable being vertical like with $y = f(x)$.
   Show: A picture.

3. We won't plot points much but the easiest way to do this is to plot x and y first then go up or down by z. Tick marks on the axes can help. A grid on the xy plane can help too. Perspective can make this a bit confusing at first. It can help to visualize a box in 3D with one corner at the origin and the other at (x, y, z). This works if they're all nonzero. Points are usually denoted by capital letters.

   Example: Plot $P = (2, 3, 5)$, $Q = (-2, 3, -1)$, $R = (0, 0, 2)$, $S = (4, 0, 0)$.
   Show: A picture.

4. Along with the three axis we get the three coordinate planes, those being the xy-plane, the yz-plane and the xz-plane. These divide 3D space into eight octants. The first octant is the one with $x, y, z \geq 0$. **PIC**

   Example: Make one up.

5. In 3D space we have a measurement of distance between $P = (x_0, y_0, z_0)$ and $Q = (x_1, y_1, z_1)$. This is denoted $|PQ|$ and is

$$|PQ| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

   Example: Make one up.

6. We also get some shapes that we'll encounter frequently:

   (a) The sphere with center (x_0, y_0, z_0) and radius r has equation

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

   Example: Make one up with picture.

   (b) The (closed) ball with center (x_0, y_0, z_0) and radius r has equation

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \leq r^2$$

   Example: Make one up with picture.