Math 241 Section 11.3: Dot Product
Dr. Justin O. Wyss-Gallifent

1. Defined the dot product: $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$.
   Example: Make one up.

2. Basic properties:

   (a) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$

   (b) $\mathbf{a} \cdot (\mathbf{b} \pm \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} \pm \mathbf{a} \cdot \mathbf{c}$

   (c) $\alpha(\mathbf{a} \cdot \mathbf{b}) = (\alpha\mathbf{a}) \cdot \mathbf{b} = \mathbf{a} \cdot (\alpha\mathbf{b})$

3. Advanced properties:

   (a) $\mathbf{a} \cdot \mathbf{b} = ||\mathbf{a}||||\mathbf{b}|| \cos\theta$ where θ is the angle between them. This follows from the Law of Cosines and is sometimes (physics especially) used as an alternate definition of the dot product.

   (b) $\mathbf{a} \perp \mathbf{b}$ iff $\mathbf{a} \cdot \mathbf{b} = 0$ and how this follows from the previous.

   (c) $\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||||\mathbf{b}||}$

   (d) $\mathbf{a} \cdot \mathbf{a} = ||\mathbf{a}||^2$ and $||\mathbf{a}|| = \sqrt{\mathbf{a} \cdot \mathbf{a}}$

4. Definition of projection and the formula

$$\text{Pr}_{\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{b} \cdot \mathbf{b}}\right)\mathbf{b}$$

   Example: Make one up.

Note: Questions like 15-17 in the homework can be confusing. All you're doing is writing the original vector as a sum of two vectors, those two vectors perpendicular to one another.