Math 241 Section 11.4: Cross Product
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1. Define the cross product. First define:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

   and then:

$$\begin{aligned} \mathbf{a} &= a_1\,\mathbf{i} + a_2\,\mathbf{j} + a_3\,\mathbf{k} \\ \mathbf{b} &= b_1\,\mathbf{i} + b_2\,\mathbf{j} + b_3\,\mathbf{k} \\ \mathbf{a} \times \mathbf{b} &= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k} \end{aligned}$$

   It's easiest to think of doing this on a case-by-case basis without writing down a messy general formula. They may have seen other definitions and that's fine too.
   Example: Make one up.

2. Basic properties:
   (a) $\mathbf{a} \times (\mathbf{b} \pm \mathbf{c}) = \mathbf{a} \times \mathbf{b} \pm \mathbf{a} \times \mathbf{c}$
   (b) $\alpha(\mathbf{a} \times \mathbf{b}) = (\alpha\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (\alpha\mathbf{b})$
   (c) $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$ (Note the negative in this one!)

   Anti-commutative, distributive over $\pm$ on both ends, associative with scalar multiplication.

3. Advanced properties:
   (a) $||\mathbf{a} \times \mathbf{b}|| = ||\mathbf{a}||||\mathbf{b}|| \sin\theta$.
   (b) $\mathbf{a}$ is parallel to $\mathbf{b}$ iff $\mathbf{a} \times \mathbf{b} = \mathbf{0}$.
   (c) The vector $\mathbf{a} \times \mathbf{b}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ via the right-hand rule. This will be one of the most useful properties of the cross product.