Math 241 Section 11.5: Equations of Lines
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1. Equations of lines are not easy; no sense of slope etc. from which to build an equation. Instead we’ll construct lines three different ways, all of which have their own use.

2. Parametric form: If \((x_0, y_0, z_0)\) is a point on the line and \(\mathbf{L} = a \mathbf{i} + b \mathbf{j} + c \mathbf{k}\) is a direction vector (the direction the line goes) then the parametric equations are

\[
\begin{align*}
x &= x_0 + at \\
y &= y_0 + bt \\
z &= z_0 + ct
\end{align*}
\]

form the other points for all possible real numbers \(t\). Emphasized how each point corresponds to a \(t\)-value and each \(t\) gives a point.
Example: When \((x_0, y_0, z_0)\) and \(\mathbf{L} = a \mathbf{i} + b \mathbf{j} + c \mathbf{k}\) are both explicitly given.

3. Changed this to vector form

\[
\mathbf{r} = \mathbf{r}(t) = (x_0 + at) \mathbf{i} + (y_0 + bt) \mathbf{j} + (z_0 + ct) \mathbf{k}
\]

and how this written like a vector but we think of it like a point. In other words we can think of it as a vector which points from the origin to the points on the line. This is actually the primary way we’ll see lines later in the course.
Example: Rewrite the previous.

4. Developed the symmetric forms by solving the parametric forms for \(t\) and setting them equal.
Example: Rewrite the previous.
Example. Did one where one of \(a, b, c\) is 0. In this case the variable with no \(t\) is left alone and the other two are solved for \(t\) and set equal.
Example. Did one where two of \(a, b, c\) are 0. In this case the two with no \(t\) are left alone and the other isn’t mentioned because the variable can be anything.

5. Distance formula from point to line. If a line has point \(P\) and direction vector \(\mathbf{L}\) then the distance from the line to another point \(Q\) equals:

\[
distance = \frac{||\mathbf{PQ} \times \mathbf{L}||}{||\mathbf{L}||}
\]

Example: Make one up.

Trickier examples:

- Finding the equation of a line when two points are given, since \(\mathbf{L}\) must be found first, and either point can be used.
- Finding where a line intersects a sphere, for example, by finding the parametric equations and plugging them into the sphere equation and solving for \(t\).
- Doing a distance from point-to-line problem when the line is given as a confusing symmetric equation since this involves extracting the necessary information from the equation.