Math 241 Section 11.5: Equations of Lines
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1. Equations of lines are not easy; no sense of slope etc. from which to build an equation. Instead we'll construct lines three different ways, all of which have their own use.

2. Parametric form: If (x_0, y_0, z_0) is a point on the line and $\mathbf{L} = a\,\mathbf{i} + b\,\mathbf{j} + c\,\mathbf{k}$ is a direction vector (the direction the line goes) then the *parametric equations* are

$$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}$$

   form the other points for all possible real numbers t. Emphasized how each point corresponds to a t-value and each t gives a point.
   Example: When (x_0, y_0, z_0) and $\mathbf{L} = a\,\mathbf{i} + b\,\mathbf{j} + c\,\mathbf{k}$ are both explicitly given.

3. Changed this to vector form

$$\mathbf{r} = \mathbf{r}(t) = (x_0 + at)\,\mathbf{i} + (y_0 + bt)\,\mathbf{j} + (z_0 + ct)\,\mathbf{k}$$

   and how this written like a vector but we think of it like a point. In other words we can think of it as a vector which points from the origin to the points on the line. This is actually the primary way we'll see lines later in the course.
   Example: Rewrite the previous.

4. Developed the symmetric forms by solving the parametric forms for t and setting them equal.
   Example: Rewrite the previous.
   Example. Did one where one of a, b, c is 0. In this case the variable with no t is left alone and the other two are solved for t and set equal.
   Example. Did one where two of a, b, c are 0. In this case the two with no t are left alone and the other isn't mentioned because the variable can be anything.

5. Distance formula from point to line. If a line has point P and direction vector $\mathbf{L}$ then the distance from the line to another point Q equals:

$$\text{distance} = \frac{||\overrightarrow{PQ} \times \mathbf{L}||}{||\mathbf{L}||}$$

   Example: Make one up.

Trickier examples:

- Finding the equation of a line when two points are given, since $\mathbf{L}$ must be found first, and either point can be used.

- Finding where a line intersects a sphere, for example, by finding the parametric equations and plugging them into the sphere equation and solving for t.

- Doing a distance from point-to-line problem when the line is given as a confusing symmetric equation since this involves extracting the necessary information from the equation.