Math 241 Section 11.6: Equations of Planes
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1. Development: Start with a point $P = (x_0, y_0, z_0)$ and a vector $\mathbf{N} = a\,\mathbf{i} + b\,\mathbf{j} + c\,\mathbf{k}$. The plane will pass through P and be perpendicular to $\mathbf{N}$. The vector $\mathbf{N}$ is called the *normal vector* for this reason, the word *normal* often means *perpendicular* in mathematics. Then if $R = (x, y, z)$ is another point on the plane then $\mathbf{N} \cdot \overrightarrow{PR} = 0$ which gets us

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

   We can then simplify this to $ax + by + cz = d$. Note how even in the simplified version the a, b, c are still the coefficients of x, y, z if they're on the same side.
   Example: Make one up.

2. Sketching planes:

   (a) None of $a, b, c = 0$: Find and plot x, y, z intercepts and draw a little triangle. Note that the plane extends from the triangle but it gives a good idea of what's going on.
   Example: Make one up.

   (b) Two of $a, b, c = 0$: Get parallel to either xy, yz or xz-plane. Think of these as shifting these planes.
   Example: Make one up.

   (c) One of $a, b, c = 0$: Draw the corresponding line in the xy, yz or xz-plane and "extend" it to a plane.
   Example: $3x - 6y = 18$ we'd plot the line in the xy-plane and then extend it in the z-direction because z can be anything.
   Example: $2x + 3z = 6$ we'd plot the line in the xz-plane and then extend it in the y-direction because y can be anything.
   Example: $2y + 2z = 8$ we'd plot the line in the yz-plane and then extend it in the x-direction because x can be anything.

3. Distance formula from point to plane. If a plane has point P and normal vector $\mathbf{N}$ and if Q is another point then the distance between Q and the plane is:

$$\text{distance} = \frac{|\mathbf{N} \cdot \overrightarrow{PQ}|}{||\mathbf{N}||}$$

   Example: Make one up.

Note: The following problems can be tricky at first:

- Finding the equation of a plane given other than P and $\mathbf{N}$. For example given three points, or given one point and a line in the plane, or given one point and a line perpendicular to the plane.
- Problems that combine 11.5 and 11.6, for example find the equation of the line formed by the intersection of two planes.