Math 241 Section 12.1: Basics of Vector Valued Functions
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1. Define a VVF as a function where a number (a parameter, typically t) goes in and a vector comes out. Typical notation:

   $$\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$$

   often with a range of t given and in 2D simply withouth the $\mathbf{k}$ component. We usually treat the vector as a point to describe location of an object at a time t. For example the vector equation of a line is a VVF.

2. To graph these we picture the vectors as anchored at the origin and we plot the endpoint. I did examples similar to the following. Note that we're not going to ask the students to draw many of these but having an idea of what the graphs look like will be extremely helpful for future problems.
   Examples: Such as the following...
   Example: $\mathbf{r}(t) = (1+2t)\,\mathbf{i} + (3-t)\,\mathbf{j}$ with $0 \leq t \leq 2$ in 2D because it's familiar - a line!
   Example: $\mathbf{r}(t) = \cos(t)\,\mathbf{i} + \sin(t)\,\mathbf{j}$ with $0 \leq t \leq \pi$ in 2D.
   Example: The above with different ranges of t.
   Example: $\mathbf{r}(t) = \cos(2t)\,\mathbf{i} + \sin(2t)\,\mathbf{j}$ with $0 \leq t \leq \pi/2$ in 2D to point out how this and the previous example have the same picture.
   Example: $\mathbf{r}(t) = \cos(t)\,\mathbf{i} + \sin(t)\,\mathbf{j} + 2\,\mathbf{k}$ with $0 \leq t \leq \pi$ in 3D.
   Example: $\mathbf{r}(t) = (2+\cos t)\,\mathbf{i} + 0\,\mathbf{j} + (3+\sin t)\,\mathbf{k}$ with $0 \leq t \leq \pi$.
   Example: $\mathbf{r}(t) = \cos(t)\,\mathbf{i} + \sin(t)\,\mathbf{j} + t\,\mathbf{k}$ with $t \geq 0$ it's a half-helix spiraling upwards.
   Example: $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j}$ with $-2 \leq t \leq 1$ in order to draw the function $y = x^2$ between $x = -2$ and $x = 1$.
   Example: $\mathbf{r}(t) = t^2\,\mathbf{i} + e^t\sin(t)\,\mathbf{j} + t\cos(t)\,\mathbf{k}$ just to point out that often we have no idea what these look like.

Note: We can use the VVF to determine when and where an object hits something. If an object whose position is described by a VVF then we can know when the object hits a plane (for example) by plugging $x(t)$, $y(t)$ and $z(t)$ into the plane equation and solving for t. We can then figure out where it hit the plane by plugging that t back into the VVF. This works for objects other than planes, too, anything with an equation.