Math 241 Section 12.3: Derivatives and Integrals of VVFs
Dr. Justin O. Wyss-Gallifent

1. Define the derivative of $\mathbf{r}(t)$ with a limit (just to make the point) but said that in practice we just take the derivatives of the components.
   Example: Make one up.
2. Application: If $\mathbf{r}(t)$ gives position of an object then:
   (a) $\mathbf{v}(t) = \mathbf{r}'(t)$ is the velocity (vector) which is tangent to the curve in the direction of the curve.
   (b) $s(t) = ||\mathbf{v}(t)||$ is the speed.
   (c) $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$ is the acceleration vector which indicates how $\mathbf{r}(t)$ is changing.

   Example: $\mathbf{r}(t) = t\,\mathbf{i} + t^3\,\mathbf{j}$ at $t = 1$ with pictures to help explain.
3. Integrals of VVFs.
   (a) Indefinite integrals of VVF: We take the individual integrals and I talked about the $+\mathbf{C}$ at the end rather than giving each components its own constant.
   Example: An application where $\mathbf{a}(t)$, $\mathbf{v}(0)$ and $\mathbf{r}(1)$ are given and we calculate $\mathbf{r}(t)$ by integrating backwards and finding the constants.
   (b) Definite Integrals of VVFs: We just take the individual integrals. We will never do this in practice.
4. Commented on the sum,difference and dot and cross-product rules with derivatives but did not emphasize that much because we don't use them that much.