1. A space curve (a curve) is the range of a VVF. Really it’s just the curve we draw. A curve can be discussed without explicitly giving an \( r(t) \). A choice of \( r(t) \) is defined as a parametrization of the curve and one curve can have many parametrizations.

Example: A line.
Example: A circle.

2. Associated Definitions

(a) Closed:
- A *closed parametrization* is a parametrization with the property that \( r(a) = r(b) \) (it starts where it ends) but otherwise does not cross itself infinitely many times.
  
Example: \( r(t) = \cos t \mathbf{i} + \sin t \mathbf{j} \) for \( 0 \leq t \leq 2\pi \).

Example: If \( t \) goes any further like twice around then the curve crosses itself infinitely many times and that’s a no-no.

- A curve is *closed* if it has a closed parametrizations.
  
Example: The circle again. There are lots of parametrizations of the circle but since there is one that’s closed then the curve is closed.

(b) Smooth:
- A *smooth parametrization* has \( r'(t) \neq 0 \) except it is permitted to be \( 0 \) at the endpoints, if it has endpoints. A good analogy is an ideal commute where your velocity is (probably) \( 0 \) at the start and end but you never need to stop on the way.

- A curve is *smooth* if it has a smooth parametrization.

(c) Piecewise Smooth:
- A *piecewise smooth* parametrization is a parametrization in which you can break the \( t \)-range into pieces on which the parametrization is smooth. Your commute is probably more like this.

- A curve is *piecewise smooth* if it has a piecewise smooth parametrization.

Example: \( r(t) = (2t + 1) \mathbf{i} + (3 - t) \mathbf{j} + t \mathbf{k} \) for all \( t \). Here \( r'(t) = 2 \mathbf{i} - 1 \mathbf{j} + 1 \mathbf{k} \) which is never \( 0 \) hence smooth.

Example: \( r(t) = t^2 \mathbf{i} + t^3 \mathbf{j} + 2 \mathbf{k} \) on \([0,5]\). Here \( r'(t) = 2t \mathbf{i} + 3t^2 \mathbf{j} + 0 \mathbf{k} \) which is \( 0 \) at \( t = 0 \) but that’s an endpoint so it’s okay.

Example: \( r(t) \) same as above but on \([-5,5]\). Here \( t = 0 \) is no longer an endpoint so it’s not smooth. It is, however, piecewise smooth.

Example: \( r(t) = t^{1/3} \mathbf{i} \) on \([-1,1]\). Here \( r'(t) = \frac{1}{3t^{2/3}} \mathbf{i} \) which is undefined at \( t = 0 \) hence neither smooth nor piecewise smooth.

(d) Length: If \( C \) is piecewise smooth on \([a,b]\) then:

\[
\text{Length of } C = \int_a^b ||r'(t)|| \, dt
\]

Example: An easy one.

Note: Examples like 22,25 in the text are good because they require a sneaky factorization inside the square root before integration.