1. Intro:
   I discussed how sometimes we want unit vectors which are tangent to and normal to a curve, what “normal to a curve” might mean. We’ll see why soon in this section.

2. The Tangent Vector:
   Defined
   \[ \mathbf{T}(t) = \frac{\mathbf{r}'(t)}{||\mathbf{r}'(t)||} \]
   This one is usually pretty intuitive. Emphasized that it’s length 1 and points in the direction of the curve.

3. The Normal Vector:
   Defined
   \[ \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||} \]
   We do this because \( \mathbf{a}(t) \) is not normal (it contains both change in velocity in the direction of motion and perpendicular to it) and so by taking \( \mathbf{T}'(t) \) instead, since \( ||\mathbf{T}|| = 1 \), we only capture the change in direction. More formally we can see the result is normal to the curve by showing it’s perpendicular to \( \mathbf{T}(t) \):
   \[
   \mathbf{T}(t) \cdot \mathbf{T}(t) = ||\mathbf{T}|| = 1 \\
   \frac{d}{dt} (\mathbf{T}(t) \cdot \mathbf{T}(t)) = 0 \\
   2\mathbf{T}'(t) \cdot \mathbf{T}(t) = 0
   
   Since \( \mathbf{T}' \perp \mathbf{T} \) and \( \mathbf{N} \) is a multiple of \( \mathbf{T}' \) we know \( \mathbf{N} \perp \mathbf{T} \).

Example: Find \( \mathbf{T} \) and \( \mathbf{N} \) at \( (4, 2) \) on the curve \( x = y^2 \) by parametrizing as \( \mathbf{r}(t) = t^2 \mathbf{i} + t \mathbf{j} \) and working it out.

4. Tangential and normal components of acceleration:
   Acceleration breaks down into two components, one in the direction of motion and one perpendicular to it. These turn out to be multiples of \( \mathbf{T} \) and \( \mathbf{N} \) and in fact:
   \[
   \mathbf{a} = a_T \mathbf{T} + a_N \mathbf{N}
   
   where:
   - The tangential component of acceleration is: \( a_T = \frac{\mathbf{v} \cdot \mathbf{a}}{||\mathbf{v}||} \)
   - The normal component of acceleration is: \( a_N = \frac{||\mathbf{v} \times \mathbf{a}||}{||\mathbf{v}||} \)

Example: Find \( a_T \) and \( a_N \) at \( t = 1 \) for \( \mathbf{r}(t) = 2t \mathbf{i} + t^2 \mathbf{j} + 1/3t^3 \mathbf{k} \).