Math 241 Section 13.4: The Chain Rule
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1. There's lots of crap in the book for the chain rule. They can ignore most of it.

2. First I pointed out that for example if $z = x^3y + y^2$ and $x = u\sin(v)$ and $y = v\cos(u)$ then really z is a function of u and v and so $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ make sense.

   Likewise if $z = xe^{xy}$ and $x = 2t^3$ and $y = \sqrt{t}$ then really z is a function of t and so $\frac{dz}{dt}$ makes sense.

3. Method for applying the chain rule:
   Step 1: Draw a tree diagram.
   Step 2: On each branch put either a d or a ∂ depending on whether it's a regular derivative (one var) or a partial derivative.
   Step 3: Find all routes from the left side to the variable we're taking the derivative with respect to.
   Step 4: Along each path find the derivatives and multiply.
   Step 5: Add the paths.
   Step 6: Substitute in for the final variable(s).

   Example: Standard.
   Example: One where the starting function ends up being a function of one variable.
   Example: One like $w = t^2 + 1/s$ and $s = t^3 + t$ because $\frac{dw}{dt} = \frac{\partial w}{\partial s}\frac{ds}{dt} + \frac{\partial w}{\partial t}$. This example is useful because it demonstrates that (a) tree branches need not be the same length and (b) $\frac{dw}{dt}$ and $\frac{\partial w}{\partial t}$ are quite different.

4. I mentioned related rates and how the chain rule is useful. The example I did was something like: Sand falls in a conical pile at $2\pi\,\text{in}^3/\text{min}$. The radius increases at 3 in/min. How fast is the height changing when $h = 10$ and $r = 8$?