Math 241 Section 13.8: Maxima and Minima
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1. Described (informally with a picture) maxima, minima, relative maxima and minima.
   Definition: For a function f a critical point is a point where the function is defined and where either all the partial derivatives are zero or at least one partial derivative is undefined.

2. Finding relative max and min. Procedure:

   (a) Find critical points.

   (b) Find the discriminant $D(x, y) = f_x x f_y y - f_{xy}^2$

   (c) For each CP:

   - If $D(x_0, y_0) > 0$ then check $f_x x(x_0, y_0)$. If $+$ rel min. If $-$ rel max.
   - If $D(x_0, y_0) < 0$ then saddle point.

   Example: $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$

3. Finding absolute extrema for $f(x, y)$ on a closed bounded region. Procedure:

   (a) Find all critical points inside the region. Take f at these point.

   (b) Find the max and min of $f(x, y)$ on the edge. How this is done depends on f and the shape of the edge but the basic idea is to use the edge equation to change f into an equation involving one variable and then work it out intuitively.

   (c) Take the largest value and smallest value from Steps 1 and 2.

   Example: $f(x, y) = 2x^2 - 3y^2$ on $x^2 + y^2 \leq 4$.

   Example: $f(x, y) = x^2 + y^2$ on the triangle with corners $(0, 0), (5, 0), (0, 3)$.