Math 241 Section 13.9: Lagrange Multipliers
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1. Goal: To find the max and/or min of $f(x, y)$ (the objective function) subject to $g(x, y) = 0$ (the constraint equation).

2. Fact: If $f(x, y)$ attains such a max and/or min then the max/min occur where $\nabla f = \lambda \nabla g$. The reason for this is not screamingly obvious and we will not discuss it in detail. The basic idea is that it happens when the level curves are parallel, which means the gradients must be parallel.

3. Method:

   (a) Identify objective $f(x, y)$
   Identify constraint g(x,y)=0

   (b) Solve for all (x, y) satisfying the system:

   $$\begin{aligned} f_x &= \lambda g_x \\ f_y &= \lambda g_y \\ g(x, y) &= 0 \end{aligned}$$

   Note: You may also find λ while doing this. That's fine, but you don't ever need λ.

   (c) Plug each resulting (x, y) into f and identify the largest and/or smallest.

   Example: Find max and min of $f(x, y) = x + 3y$ with $x^2 + y^2 = 9$.
   Example: Find min of $f(x, y) = xy$ with $y = 2x + 4$.
   Example: Find max and min of $f(x, y) = x + xy$ with $x^2 + y^2 = 1$.