Math 241 Section 14.1: Double Integrals
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1. Reminded them that for $f(x)$ defined on $I = [a, b]$ the single integral is (signed) area under a curve. Drew pictures.
2. For $f(x, y)$ defined on R in xy-plane define the double integral
$$\int\int_R f(x, y)\, dA$$
as the (signed) volume. Picture to clarify.
3. Switch gears and do examples of iterated double integrals, basically just understanding the notation and process, ignoring relevance.
4. The point now is to rewrite $\int\int_R f(x, y)\, dA$ as an iterated double integral. This process with depend entirely on the shape of R. For now just rectangular parametrizations.
   (a) Definition: R is vertically simple if R is:
   - Between two constant x-values $x = a$ and $x = b$.
   - Between functions $y = B(x)$ and $y = T(x)$.

   Picture to clarify.
   Then:
$$\int\int_R f(x, y)\, dA = \int_a^b \int_{B(x)}^{T(x)} f(x, y)\, dy\, dx$$
   Example: $\int\int_R xy\, dA$ where R is the region between $y = \frac{1}{2}x$ and $y = x$ and to the left of $x = 6$.
   (b) Definition: R is horizontally simple if R is:
   - Between two constant y-values $y = c$ and $y = d$.
   - Between functions $x = L(y)$ and $x = R(y)$.

   Picture to clarify.
   Then:
$$\int\int_R f(x, y)\, dA = \int_c^d \int_{L(y)}^{R(y)} f(x, y)\, dx\, dy$$
   Example: $\int\int_R x + y\, dA$ where R is the region between $x = y^2$ and $y = \frac{1}{5}x$.
   (c) Definition: R is simple if it's both, and can then be done either way.