Math 241 Section 14.2: Double Integrals in Polar
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1. Intro: Sometimes R can be easier to describe using polar. A region is described in polar as being between two angles $\theta = \alpha$ and $\theta = \beta$ and between two functions $r = N(\theta)$ and $r = F(\theta)$ where N is nearer to the origin and F is further from the origin.

2. Polar functions to rememeber:

   - Circles: $r = a$, $r = a\cos\theta$, $r = a\sin\theta$
   - Cardioids: $r = a + a\cos\theta$, $r = a + a\sin\theta$
   - Converting rectangular: For example $x = 2$ becomes $r\cos\theta = 2$ or $r = 2\sec\theta$.

3. If R is described in polar then:

$$\int\int_R f(x,y)\,dA = \int_\alpha^\beta \int_{N(\theta)}^{F(\theta)} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

   Note 1: the $f(r\cos\theta, r\sin\theta)$ means convert f into polar.

   Note 2: Do not forget the additional r. We'll have a good explanation of this later.

   Example: $\int\int_R x dA$ where R is the semicircle $x^2 + y^2 \leq 9$ with $x \geq 0$.

   Example: $\int\int_R 1 dA$ where R is the region inside $r = 1 + \cos\theta$ and outside $r = 1$.

4. Note about 14.1 and 14.2: Sometimes an iterated integral has been set up one way (VS, HS, P) and is easier another way. In this case we might rewrite it.

   Example: The integral

$$\int_0^1 \int_{x\sqrt{3}}^{\sqrt{4-x^2}} \sqrt{x^2 + y^2}\,dy\,dx$$

   is particular icky. The region R however is simply the pie-slice in the first quadrant inside $r = 2$ and between $\theta = \pi/3$ and $\theta = \pi/2$. Therefore we can rewrite it as

$$\int_{\pi/3}^{\pi/2} \int_0^2 r^2\,dr\,d\theta$$

   which is much more manageable.