Math 241 Section 14.4: Triple Integrals in Rectangular
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1. Introduction: If D is a solid and $f(x, y, z)$ is the density around (x, y, z) then $\int\int\int_D f(x, y, z)\, dV$ represents the mass of D. This idea applies to any density sort of thing, like electrical charge density, for example. The question is how to evaluate this.

2. This depends on how D is described. For 14.4 we assume D is between $F_1(x, y)$ and $F_2(x, y)$ and above the region R in the xy-plane where R is either vertically or horizontally simple. If D is described as such then:

   (a) If R is VS then

$$\int\int\int_D f(x, y, z)\, dV = \int_a^b \int_{B(x)}^{T(x)} \int_{F_1(x,y)}^{F_2(x,y)} f(x, y, z)\, dz\, dy\, dx$$

   (b) If R is HS then

$$\int\int\int_D f(x, y, z)\, dV = \int_c^d \int_{L(y)}^{R(y)} \int_{F_1(x,y)}^{F_2(x,y)} f(x, y, z)\, dz\, dx\, dy$$

   Example: Find the mass of D where is between $z = x^2 + y^2$ and $z = 1 + x^2 + y^2$ and above R the triangle in the xy-plane with corners $(0, 0), (0, 1), (1, 0)$ and the density is $f(x, y, z) = xz$.

3. I then commented that volume is $\int\int\int_D 1\, dV$ and why.

   Example: Find the volume of D the wedge under $x + 2y + z = 6$ and in the first octant.