Math 241 Section 14.9: Parametrization of Surfaces
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1. First a reminder about how we parametrized a curve $\mathbf{r}(t)$. That is, for various t we think of $\mathbf{r}(t)$ as a point on the curve and for all t we get all points.

   The idea for a surface is that given a surface Σ we want to give a parametrization $\mathbf{r}(u, v)$ for a range of u and $v$so that as those two variables run over their ranges we get all the points on the surface.

   Typically we don't use u and v (in fact we never do) but those are stand-ins for more common variables like x, y, z, r, θ, ϕ, and ρ.

2. Examples. Emphasize why we make choices of the two variables the way we do. This can often be confusing at first. Emphasize that the choice of variables is often based on the restriction rather than the surface.

   Example: A small rectangle at $z = 2$ with $-1 \leq x \leq 1$ and $-2 \leq y \leq 2$:
   $\mathbf{r}(x, y) = x\,\mathbf{i} + y\,\mathbf{j} + 2\,\mathbf{k}$ with $-1 \leq x \leq 1$ and $-2 \leq y \leq 2$.

   Example: If we fix $y = 3$ instead then we can get something like:
   $\mathbf{r}(x, z) = x\,\mathbf{i} + 3\,\mathbf{j} + z\,\mathbf{k}$ with $0 \leq x \leq 1$ and $0 \leq z \leq 5$.

   Example: The disk of radius 2 at $z = 3$ centered on the z-axis:
   $\mathbf{r}(r, \theta) = r\cos\theta\,\mathbf{i} + r\sin\theta\,\mathbf{j} + 3\,\mathbf{k}$ with $0 \leq \theta \leq 2\pi$ and $0 \leq r \leq 2$.

   Example: Change the previous one so x is fixed:
   $\mathbf{r}(y, z) = ...$

   Example: The part of $z = 16 - x^2 - y^2$ above a rectangle in the xy-plane.
   $\mathbf{r}(x, y) = ...$

   Example: The cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 5$:
   $r(z, \theta) = 3\cos\theta\,\mathbf{i} + 3\sin\theta\,\mathbf{j} + z\,\mathbf{k}$ with $0 \leq \theta \leq 2\pi$ and $0 \leq z \leq 5$.

   Example: The hemisphere $x^2 + y^2 + z^2 = 9$ above the xy-plane.
   $\mathbf{r}(\theta, \phi) = ...$

NOTE: There is a document on canvas with a bunch of extra examples (the book is sorely lacking) as well as solutions.
NOTE: All the examples I did today have constant bounds on the two variables. Soon enough I'll introduce some which have functional bounds on one of them.