Math 241 Section 15.3: The Fundamental Theorem of Line Integrals
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1. Reminder about how the Fundamental Theorem of Calculus works. It sometimes helps students see the analogy. Plus this analogy arises later in other theorems.

2. FTOLI: If $\mathbf{F}$ is conservative with potential function f then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(\text{endpoint of } C) - f(\text{startpoint of } C)$$

   Example: Draw a really awful curve in 2D but make the endpoints clear.

   Example: Give $\mathbf{r}(t)$ so we have to find the endpoints via $\mathbf{r}$ in that case.

3. Notes:

   (a) $\mathbf{F}$ MUST BE CONSERVATIVE!!!

   (b) If $\mathbf{F}$ is conservative and C is closed then $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$.

   (c) These problems can also appear with $\int_C M\,dx + N\,dy + P\,dz$ notation.

   (d) If $\mathbf{F}$ is conservative then we say the integral is independent of path.