Math 241 Section 15.7: Stokes' Theorem
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1. Introduction: Stokes' Theorem can be thought of as a version of Green's Theorem which applies to surface in 3D space, although it would be more accurate to say that Green's Theorem is a version of Stokes' Theorem when the surface is in the xy-plane.

2. Induced Orientations: If C is the edge of Σ and if C has an orientation (direction) then C induces an orientation on Σ using the right-hand rule; if your fingers follow C's orientation then your thumb points in an orientation for Σ. We say Σ has the induced orientation from R.

   This also works in reverse but we will not need it this way.

3. Stokes' Theorem: Suppose C is the edge of Σ and C is oriented. Then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int \int_\Sigma (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$$

   where Σ has the induced orientation from R.

   Note: Stokes' Theorem leads to a multiple-step procedure:

$$\int_C \underset{\text{Stokes'}}{\longrightarrow} \int \int_\Sigma \underset{\text{Param }\Sigma}{\longrightarrow} \pm \int \int_R \underset{\text{Use Ineq}}{\longrightarrow} \int_*^* \int_*^* \underset{\text{Grunt}}{\longrightarrow} \text{Value}$$

   So be clear about showing all necessary steps.

   Example: Suppose C is the curve on the surface $z = 9 - x^2$ lying above the triangle with vertices $(0,0,0)$, $(2,0,0)$, $(0,1,0)$ and having counterclockwise orientation when viewed from above. Evaluate $\int_C 2y\, dx + xz\, dy + z\, dz$.

   Example: Suppose C is the intersection of $x^2 + z^2 = 16$ with $y = 9 - x^2$ having clockwise orientation when viewed from the positive y-axis. Evaluate $\int_C (x^2\,\mathbf{i} + xy\,\mathbf{j} + xz\,\mathbf{k}) \cdot d\mathbf{r}$.