MATH 246 Groupwork 2.1 & 2.3 **Name:** _____

1. Put the following linear differential equations in normal form and determine the interval of existence (and uniqueness) for the solution to the associated initial value problem.

   (a) $(t-1)y'' + 2y' + \left(\frac{t-1}{t^2-10t}\right) y = \sqrt{20-t}$ with $y(12) = 7$ and $y'(12) = -1$.

   (b) $(t-1)y'' + 2y' + \left(\frac{t-1}{t^2-10t}\right) y = \sqrt{20-t}$ with $y(7) = 12$ and $y'(7) = 0$.

   (c) $y''' + \ln(-t) = \sqrt{t+2}y' + ty$ with $y(-1) = 0$, $y'(-1) = 60$ and $y''(-1) = \pi$.

2. For each of the following linear systems, if the system is homogeneous determine if it could have nontrivial solutions and if the system is nonhomogeneous determine if it has a single solution or not. Use determinants only.

   (a) The system:

   $$\begin{aligned} x + 3y &= 3 \\ 3x - 2y &= 0 \end{aligned}$$

   (b) The system

   $$\begin{aligned} -x + 6y &= 0 \\ x - 6y &= 1 \end{aligned}$$

(c) The system

$$\begin{aligned} -2x + 6y &= 0 \\ 3x + 6y &= 0 \end{aligned}$$

(d) The system

$$\begin{aligned} 2x + 3y + z &= 2 \\ x - 5y &= 1 \\ -x + 8y + 3z &= 10 \end{aligned}$$

(e) The system

$$\begin{aligned} 2x + y - 3z &= 0 \\ 5x + 2y - 2z &= 0 \\ 4x + y + 5z &= 0 \end{aligned}$$

3. Consider the IVP:

$$y'' - t^2 \sin(t^3) \cos(t^3) y' - \sin(t^3) y = 0 \text{ with } y(0) = 0 \text{ and } y'(0) = 0$$

Consider the function $f(t) = \sin(t^3)$. Without plugging it into the differential equation, determine why this function could not be a solution to the IVP.

Hint: Find $f(0)$ and $f'(0)$, then think about what definitely is a solution to the IVP. Conclude from there.