MATH 246 Groupwork 3.9 **Name:** ____________________

1. Suppose an autonomous system of differential equations is conservative and has the following stationary points with eigenpairs given.

$$
\begin{aligned}
(-2,0) &: \left\{2, \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}, \left\{4, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\} \\
(2,0) &: \left\{-1, \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}, \left\{-3, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\} \\
(0,2) &: \left\{-1, \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right\}, \left\{1, \begin{bmatrix} 2 \\ -1 \end{bmatrix}\right\}
\end{aligned}
$$

(a) Sketch a feasible phase portrait. Emphasize the solution with initial value $\bar{x}(0) = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$.

2. Consider the system of differential equations

$$\begin{aligned} x' &= y \\ y' &= -4x + x^2 \end{aligned}$$

(a) Find the stationary points.

(b) For each stationary point, find the coefficient matrix of the linearization. Find the eigenvalues and if necessary the eigenvectors for each.

(c) Sketch a feasible phase portrait.