MATH 246 Homework 3.1
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## Directions:

- Work should be done neatly and on separate paper.
- Enough work must be shown so that the steps you are taking is clear.

1. Show that the pair $x_{1}(t)=\cos (2 t)+\sin (2 t)$ and $x_{2}(t)=\sin (2 t)$ form a solution to the system

$$
\begin{aligned}
x_{1}^{\prime} & =2 x_{1}-4 x_{2} \\
x_{2}^{\prime} & =2 x_{1}-2 x_{2}
\end{aligned}
$$

2. Rewrite $2 y^{\prime \prime}+t y^{\prime}-y=\sin t$ with $y(0)=1, y^{\prime}(0)=-1$ as a first-order system with initial value.
3. Rewrite $y^{\prime \prime \prime}+2 y^{\prime \prime}-y^{\prime}+3 y=1$ as a first-order system with three variables.
4. Tank 1 has volume of 200 Liters and Tank 2 has volume 100 Liters. Initially both are full with Tank 1 containing salt at $2 \mathrm{~g} / \mathrm{L}$ and Tank 2 containing salt at $3 \mathrm{~g} / \mathrm{L}$ The Tank 1 mixture is flowing from Tank 1 to Tank 2 at $5 \mathrm{~L} / \mathrm{min}$ while the Tank 2 mixture is flowing from Tank 2 to Tank 1 at $3 \mathrm{~L} / \mathrm{min}$. Fresh water is flowing into Tank 1 at $6 \mathrm{~L} / \mathrm{min}$ while the Tank 1 mixture flows out to a drain at $4 \mathrm{~L} / \mathrm{min}$. Water at $4 \mathrm{~g} / \mathrm{L}$ is flowing into Tank 2 at $7 \mathrm{~L} / \mathrm{min}$ while the Tank 2 mixture flows out to a drain at $9 \mathrm{~L} / \mathrm{min}$. Let $x_{1}$ represent the amount of salt in Tank 1 at time $t$ and $x_{2}$ represent the amount of salt in Tank 2 at time $t$. Draw a tank picture for this situation and write down the corresponding system with initial values.
