## MATH 246 Homework 3.1 Justin Wyss-Gallifent

## Directions:

- Work should be done neatly and on separate paper.
- Enough work must be shown so that the steps you are taking is clear.
- 1. Show that the pair  $x_1(t) = \cos(2t) + \sin(2t)$  and  $x_2(t) = \sin(2t)$  form a solution to the system

$$\begin{aligned} x_1' &= 2x_1 - 4x_2 \\ x_2' &= 2x_1 - 2x_2 \end{aligned}$$

- 2. Rewrite  $2y'' + ty' y = \sin t$  with y(0) = 1, y'(0) = -1 as a first-order system with initial value.
- 3. Rewrite y''' + 2y'' y' + 3y = 1 as a first-order system with three variables.
- 4. Tank 1 has volume of 200 Liters and Tank 2 has volume 100 Liters. Initially both are full with Tank 1 containing salt at 2 g/L and Tank 2 containing salt at 3 g/L The Tank 1 mixture is flowing from Tank 1 to Tank 2 at 5 L/min while the Tank 2 mixture is flowing from Tank 1 to Tank 2 at 5 L/min while the Tank 2 mixture is flowing from Tank 1 at 3 L/min. Fresh water is flowing into Tank 1 at 6 L/min while the Tank 1 mixture flows out to a drain at 4 L/min. Water at 4 g/L is flowing into Tank 2 at 7 L/min while the Tank 2 mixture flows out to a drain at 9 L/min. Let  $x_1$  represent the amount of salt in Tank 1 at time t and  $x_2$  represent the amount of salt in Tank 2 at time t. Draw a tank picture for this situation and write down the corresponding system with initial values.