MATH 246 Homework 3.5
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Directions:

- Work should be done neatly and on separate paper.
- Enough work must be shown so that the steps you are taking is clear.

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1. Consider the system:
$$\bar{x}' = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \bar{x}$$
A fundamental matrix for this system is:
$$\Psi = \begin{bmatrix} -3e^t & -e^{-t} \\ e^t & e^{-t} \end{bmatrix}$$
   (a) Find the natural fundamental matrix Φ associated to $t_I = \ln(2)$.
   (b) Use your answer to (a) to find the solutions to the two IVPs:
$$\bar{x}(\ln 2) = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \text{ and } \bar{x}(\ln 2) = \begin{bmatrix} 0 \\ 42 \end{bmatrix}$$

2. Consider the second order linear differential equation $D^2y - 3Dy = 0$.
   (a) Find a fundamental pair using Chapter 2 methods.
   (b) Rewrite this differential equation as a system of two first-order differential equations using $x_1 = y$ and $x_2 = y'$.
   (c) Using this rewrite, the fundamental pair from (a) will then give you a fundamental pair for the system. Write down this fundamental pair.
   (d) Find the natural fundamental matrix Φ associated to $t_I = 0$.
   (e) Use your answer to (c) to find the solution to the IVP with:
$$\bar{x}(0) = \begin{bmatrix} 17 \\ 42 \end{bmatrix}$$

3. Find the general solution to each of the following. Then for those with an initial value, find the specific solution.
   (a) $\bar{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \bar{x}$
   (b) $\bar{x}' = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \bar{x}$
   (c) $\bar{x}' = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} \bar{x}$ with $\bar{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
   (d) $\bar{x}' = \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix} \bar{x}$
   (e) $\bar{x}' = \begin{bmatrix} 3 & 0 \\ 4 & 3 \end{bmatrix} \bar{x}$
   (f) $\bar{x}' = \begin{bmatrix} 5 & 4 \\ -25 & -15 \end{bmatrix} \bar{x}$ with $\bar{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$
   (g) $\bar{x}' = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \bar{x}$