**MATH 246 Homework 3.9**
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**Directions:**

- Work should be done neatly and on separate paper.
- Enough work must be shown so that the steps you are taking is clear.

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1. Suppose an autonomous system of differential equations is conservative and has the following stationary points with eigenpairs given.

$$\begin{aligned}
&(0,0) : \{-1 \pm 2i, ?\} \\
&(2,0) : \left\{-2, \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right\}, \left\{3, \begin{bmatrix} 1 \\ -3 \end{bmatrix}\right\} \\
&(4,0) : \{\pm i, ?\}
\end{aligned}$$

   (a) Sketch a feasible phase portrait.

   (b) Separately sketch the solution satisfying $\bar{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

2. Consider the system of differential equations

$$\begin{aligned}
x' &= x + y \\
y' &= x^2 - y
\end{aligned}$$

   (a) Find the stationary points.

   (b) For each stationary point, find the coefficient matrix of the linearization. Find the eigenvalues and if necessary the eigenvectors for each.

   (c) Sketch a feasible phase portrait.

   (d) On a separate axes, sketch the solution to the initial value problem with $\bar{x}(0) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$.

3. Consider the system of differential equations

$$\begin{aligned}
x' &= xy - y \\
y' &= -\frac{1}{2}y^2 + 2x
\end{aligned}$$

   (a) Find the stationary points.

   (b) For each stationary point, find the coefficient matrix of the linearization. Find the eigenvalues and if necessary the eigenvectors for each.

   (c) Sketch a feasible phase portrait.

   (d) On a separate axes, sketch the solution to the initial value problem with $\bar{x}(0) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.