MATH 246: Chapter 2 Section 1: Intro to Higher Order Linear
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Main Topics:

- Reminder and Notation.
- Interval of Existence.

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1. **Introduction** Since higher order DEs are difficult we're going to focus on linear higher order DEs. We'll narrow it down even more but for now that's where we are. Just a reminder that these look like, all in *linear normal form*:

First-Order	$y' + a(t)y = f(t)$ (We can solve these)
Second-Order	$y'' + a(t)y' + b(t)y = f(t)$
Third-Order	$y''' + a(t)y'' + b(t)y' + c(t)y = f(t)$
Etc.	Etc.

2. **Notation Note**

   It's not uncommon to see an alternate notation for the derivative from here on. We often write Dy instead of y', D^2y instead of y'' and so on.

3. **Existence Theory**

   The theory is similar to what we've seen for first-order but the initial value needs a bit more:

   - A first order linear IVP requires knowing $y(t_I) = y_0$. There is a unique solution on the *interval of existence* which is the largest open interval containing t_I on which $a(t)$ and $f(t)$ are differentiable.
   - A second order linear IVP requires knowing $y(t_I) = y_0$ and $y'(t_I) = y_1$. There is a unique solution on the *interval of existence* which is the largest open interval containing t_I on which $a(t)$, $b(t)$ and $f(t)$ are differentiable.
   - A third order linear IVP requires knowing $y(t_I) = y_0$ and $y'(t_I) = y_1$ and $y''(t_I) = y_2$. There is a unique solution on the *interval of existence* which is the largest open interval containing t_I on which $a(t)$, $b(t)$, $c(t)$ and $f(t)$ are differentiable.
   - From here you can certainly see the pattern.

   Note: The proof (of existence and uniqueness) is difficult. The special case for first order linear is easy and we saw it because we explicitly constructed the solution.

   Example: $y'' + \frac{1}{t}y' - \frac{1}{t-3}y = t$ with $y(1) = 17$ and $y'(1) = 2$ has a unique solution on $(0, 3)$. If instead we have $y(4) = 17$ then this has a unique solution on $(3, \infty)$.

   Example: For $t^{-1/2}y'' + e^t y' - \sin(t)y = \frac{t}{6-t}$ with $y(1) = 8$ and $y'(1) = 3$ we have to first rewrite in linear normal form as $y'' + e^t\sqrt{t}y' - \sqrt{t}\sin(t)y = \frac{t^{3/2}}{6-t}$ which then has a unique solution on $(0, 6)$.

   Example: The IVP $y''' - \frac{1}{t}y'' + e^t y' - \sin(t)y = \frac{t}{10-t}$ with $y(3) = 8$ and $y'(3) = 3$ and $y''(3) = 5$ has a unique solution on $(0, 10)$.

   Example: The IVP $D^2y - Dy - 2y = 0$ with $y(0) = 1$ and $y'(0) = -3$ has a unique solution on $(-\infty, \infty)$. If we notice that $y = e^{2x}$ is a solution then we know it's the only solution.