1. For the statement:

\[ \forall x, y \in \mathbb{Z} \text{ if } x \text{ and } xy \text{ are odd then } y \text{ is odd.} \]

For each of - direct proof, proof by contrapositive and proof by contradiction - write down what you would assume and what you would need to prove. Determine which is easier, explain why, and then provide a proof.

2. In what way is disproving a \( \forall \) statement the same as proving a \( \exists \) statement?

3. Prove: For \( f(x) = x^8 + 4x^2 + 8x \) prove \( \exists x \in \mathbb{R} \) such that \( f'(x) = 0. \)

4. Disprove: \( \forall n \in \mathbb{Z} \text{ if } n^2 + 3n \text{ is even then } n^2 \text{ is odd.} \)

5. Disprove: \( \exists a, b \in \mathbb{Z} \text{ with } a, b \text{ odd such that } 4 \mid (3a^2 + 7b^2). \)