

MATH310 Summer 2022 Exam 1

NAME:

1. For each of the following, determine first if the item is a proposition or not. If it is a proposition [10 pts] determine if it is true or false. If it is not a proposition leave the last column blank.

Item	Proposition Y/N	T/F
$\forall x \in \mathbb{R}, x^2 + 1 > -100$		
All nonnegative real numbers are positive.		
All positive real numbers are nonnegative.		
$\{\} \subseteq \emptyset$		
$2x$		

2. Write down a useful negation of the sentence:

[5 pts]

If I am not happy then I buy chocolate or I buy Doritos.

Solution:

3. Consider the following sentence:

If $x^2 > 9$ then either $x < -3$ or $x > 3$.

(a) Write down the converse.

[5 pts]

Solution:

(b) Write down the contrapositive.

[5 pts]

Solution:

4. The following is the definition of a Cauchy sequence:

[10 pts]

$$\forall \epsilon > 0, \exists N \in \mathbb{Z}^+, \forall n, m \geq N, (n, m \geq N \rightarrow |a_n - a_m| < \epsilon)$$

Negate this statement.

Solution:

5. Give examples of sets A , B , C such that $A \subseteq B$, $B \subseteq C$ and $A \in C$.

[8 pts]

Solution:

6. Let $S = \{1, 2, 3, 4, 5\}$. Describe the set $\{3, 5, 7, 9\}$ in the form $\{f(x) \mid x \in S \text{ and } p(x)\}$ for some function $f(x)$ and open sentence $p(x)$.

[7 pts]

Solution:

7. Use a truth table to determine if $P \rightarrow (Q \wedge P) \not\equiv (P \rightarrow Q) \wedge Q$.

[10 pts]

Solution:

8. Prove that:

$$\{x \in \mathbb{R} \mid |x + 3| = 5 - |x|\} = \{-4, 1\}$$

[10 pts]

Solution:

9. Prove directly that $\forall x \in \mathbb{R}, |2 - x| - x \geq -2$.

[10 pts]

Solution:

10. Prove that $\forall a, b, c \in \mathbb{Z}$, if a divides b and a divides $2b + c$ then a divides c .

[10 pts]

Solution:

11. Prove by contradiction that $\forall a, b \in \mathbb{Z}$ if a and $a - 2b$ are odd, then a is odd. You must use the definitions of odd and even for this. [10 pts]

Solution: