MATH310 Exam 1 Sample Questions

1. Which of the following are sets?
   (a) $\{\{\{\}\}\}$
   (b) $\{\mathbb{Z}, \emptyset\}$
   (c) $\mathbb{Z}, \mathbb{R}$
   (d) $\{1, 2, 3, ...\}$
2. For each of the following sets A and B determine if A is a subset of B. If not, find an element in A which is not in B.
   (a) $A = \{5, 6\}$ and $B = \{4, 5, 6\}$
   (b) $A = \{3x \mid x \in \mathbb{Z}\}$ and $B = \{6x \mid x \in \mathbb{Z}\}$
3. Give examples of sets A, B, C such that $A \subseteq B$, $B \subseteq C$ and $A \in C$.
4. Let $S = \{1, 2, 3, 4, 5\}$. Describe the set $\{3, 5, 7, 9\}$ in the form $\{f(x) \mid x \in S \text{ and } p(x)\}$ for some function $f(x)$ and open sentence $p(x)$.
5. Use a truth table to show that $P \to (Q \wedge R) \not\equiv (P \to Q) \wedge R$.
   Note: You don't have to draw every row of the truth table, just enough rows get the job done.
6. Consider the open sentences over the domain $\mathbb{R}$

   $$P(x) : x - 1 \geq 0 \text{ and } Q(x) : x^2 - 9 \geq 0$$

   Find all x such that $P(x) \vee Q(x)$ is true. List as intervals.
7. Determine if the following are true or false, with justification.
   (a) $\exists x \in \{1, 2, 3\}, 5x - 1$ is divisible by 3.
   (b) $\forall x \in \mathbb{N}, 2x + 1$ is prime.
   (c) $\forall x \in \mathbb{N}, \exists y \in \mathbb{R}, y^2 = x$.
8. Prove $\forall x \in \mathbb{R}$, $|2 - x| - x \geq -2$.
9. Prove $\forall x, y \in \mathbb{Z}$, $x + y$ is odd if and only if x and y have opposite parity.
10. Are the following statements true or false?
    (a) $\{1\} \in \{1, 2, 3\}$
    (b) $\emptyset \in \{\}$
    (c) $\emptyset \in \mathcal{P}(\{1, 2, 3\})$
11. Write the elements in $\mathcal{P}(\mathcal{P}(\{1\}))$.
12. Let $S = \{0, 3, 6, 9, 12\}$. Describe the set $\{1, 2, 3, 4\}$ in the form $\{f(x) \mid x \in S \text{ and } p(x)\}$ for some function $f(x)$ and open sentence $p(x)$.

13. Fill in the following truth table only for the possibilities given.

P	Q	R	$P \wedge Q$	$(P \wedge Q) \to R$	$R \to (P \wedge Q)$
T	T	T			
T	F	T			
F	T	F			
F	F	F			

14. Consider the open sentences over the domain $\mathbb{R}$

$$P(x) : x - 1 \geq 0 \text{ and } Q(x) : x^2 + 3x \leq 0$$

Find all $x \in \mathbb{R}$ such that $P(x) \to Q(x)$ is true. List as intervals.

15. Prove $\forall x \in \mathbb{Z}$, a is even iff a^2 is even.

16. Prove that if $A \subseteq B$, $B \subseteq C$ and $C \subseteq A$ then $A = B$ and $B = C$.

17. Prove that:

$$\left\{ x \in \mathbb{R} \,\middle|\, |x| = 6 - |2x| \right\} = \{-2, 2\}$$