MATH310 Exam 1 Sample Questions

1. Which of the following are sets?
   (a) \{\{\}\}
   (b) \{\mathbb{Z}, \emptyset\}
   (c) \mathbb{Z}, \mathbb{R}
   (d) \{1, 2, 3,\ldots\}

2. For each of the following sets \(A\) and \(B\) determine if \(A\) is a subset of \(B\). If not, find an element in \(A\) which is not in \(B\).
   (a) \(A = \{5, 6\}\) and \(B = \{4, 5, 6\}\)
   (b) \(A = \{3x \mid x \in \mathbb{Z}\}\) and \(B = \{6x \mid x \in \mathbb{Z}\}\)

3. Give examples of sets \(A\), \(B\), \(C\) such that \(A \subseteq B\), \(B \subseteq C\) and \(A \in C\).

4. Let \(S = \{1, 2, 3, 4, 5\}\). Describe the set \(\{3, 5, 7, 9\}\) in the form \(\{f(x) \mid x \in S \text{ and } p(x)\}\) for some function \(f(x)\) and open sentence \(p(x)\).

5. Use a truth table to show that \(P \rightarrow (Q \land R) \neq (P \rightarrow Q) \land R\).
   Note: You don’t have to draw every row of the truth table, just enough rows get the job done.

6. Consider the open sentences over the domain \(\mathbb{R}\)
   \[P(x) : x - 1 \geq 0 \quad \text{and} \quad Q(x) : x^2 - 9 \geq 0\]
   Find all \(x\) such that \(P(x) \lor Q(x)\) is true. List as intervals.

7. Determine if the following are true or false, with justification.
   (a) \(\exists x \in \{1, 2, 3\}, 5x - 1\) is divisible by 3.
   (b) \(\forall x \in \mathbb{N}, 2x + 1\) is prime.
   (c) \(\forall x \in \mathbb{N}, \exists y \in \mathbb{R}, y^2 = x\).

8. Prove \(\forall x \in \mathbb{R}, |2 - x| - x \geq -2\).

9. Prove \(\forall x, y \in \mathbb{Z}, x + y\) is odd if and only if \(x\) and \(y\) have opposite parity.

10. Are the following statements true or false?
    (a) \(\{1\} \in \{1, 2, 3\}\)
    (b) \(\emptyset \in \{\}\)
    (c) \(\emptyset \in \mathcal{P}(\{1, 2, 3\})\)

11. Write the elements in \(\mathcal{P}(\mathcal{P}(\{1\}))\).

12. Let \(S = \{0, 3, 6, 9, 12\}\). Describe the set \(\{1, 2, 3, 4\}\) in the form \(\{f(x) \mid x \in S \text{ and } p(x)\}\) for some function \(f(x)\) and open sentence \(p(x)\).
13. Fill in the following truth table only for the possibilities given.

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14. Consider the open sentences over the domain \( \mathbb{R} \)

\[ P(x) : x - 1 \geq 0 \text{ and } Q(x) : x^2 + 3x \leq 0 \]

Find all \( x \in \mathbb{R} \) such that \( P(x) \rightarrow Q(x) \) is true. List as intervals.

15. Prove \( \forall x \in \mathbb{Z}, a \text{ is even iff } a^2 \text{ is even.} \)

16. Prove that if \( A \subseteq B, B \subseteq C \text{ and } C \subseteq A \) then \( A = B \) and \( B = C \).

17. Prove that:

\[ \left\{ x \in \mathbb{R} \mid |x| = 6 - |2x| \right\} = \{-2, 2\} \]