MATH310 Summer 2022 Exam 2

NAME:

Instructions: Please do all problems on the pages and in the spaces provided. This exam will be scanned into Gradescope and if your answers are not in the correct locations they will not be found or graded!

Extra Credit (Up to 2 Points): Write a haiku related to proofs. A haiku is a short poem with three lines. The first line has 5 syllables, the second line has 7 syllables, the third line has 5 syllables.

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1. Provide counterexamples which disprove each of the following statements. You do not need to prove that your counterexamples work!

   (a) $\forall x \in \mathbb{Z}$, $\exists y \in \mathbb{Z}$ such that $x + |y| = 0$. [2 pts]

   Solution:

   (b) $\forall x, y \in \mathbb{R}$ we have $x^2 + y^2 > 0$. [2 pts]

   Solution:

   (c) $\forall a, b, c \in \mathbb{Z}$ if a divides $b + c$ then a divides b or a divides c. [2 pts]

   Solution:

2. Define the relation $R \subset \mathbb{Z} \times \mathbb{Z}$ by: [12 pts]

$$R = \left\{ (x, y) \in \mathbb{Z} \times \mathbb{Z} \,\middle|\, x + 9y \text{ is divisible by } 5 \right\}$$

Prove that R is an equivalence relation.

Solution:

3. Prove using weak induction that $\forall n \geq 4$ that $3^n > 5n^2$. [10 pts]

Solution:

4. Prove using strong induction that any postage of 18 cents or more can be made using 3-cent and 10-cent stamps. [10 pts]

   Solution:

5. Suppose $A, B \subseteq U$. Prove that: [10 pts]

$$\forall x \in U,\ \mathcal{X}_{A-B}(x) = \mathcal{X}_{A\cap\overline{B}}(x)$$

Solution:

6. Define $f(x) = x$ for $x \in \mathbb{R}$ and $g(x) = |x|$ for $x \in \mathbb{R}$. Prove that: [12 pts]

$$\forall A \subseteq \mathbb{R},\ f\Big|_A = g\Big|_A \text{ iff } A \subseteq [0, \infty)$$

Solution:

7. Prove that the following function $f : \mathbb{R} - \{1\} \longrightarrow \mathbb{R}$ is invertible and find a rule for the inverse: [10 pts]

$$f(x) = \frac{2 - x}{x - 1}$$

Solution:

8. Prove or disprove that the following function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is surjective: [10 pts]

$$f(x) = 2x + 3|x|$$

Solution:

9. For sets A and B, prove that $(A - B) \cap \bar{A} = \emptyset$. [10 pts]

Solution:

10. Prove that if $f : A \to B$ and $g : B \to C$ are both injective then so is $g \circ f : A \to C$. [10 pts]

Solution: