MATH310 Summer 2022 Exam 3

NAME:

Instructions: Please do all problems on the pages and in the spaces provided. This exam will be scanned into Gradescope and if your answers are not in the correct locations they will not be found or graded!

1. Write down the following definitions:

   (a) $\{x_n\} \longrightarrow x$ [4 pts]

   Solution:

   (b) $\lim\limits_{x \to x_0} f(x) = L$ [3 pts]

   Solution:

   (c) $f(x)$ is continuous at x_0 [3 pts]

   Solution:

2. Use the definition to prove: [10 pts]

$$\left\{\frac{6n}{1-2n}\right\} \longrightarrow -3$$

Solution:

3. Use the definition to prove: [10 pts]

$$\left\{2 + \frac{1}{n}\right\} \not\to 1$$

Solution:

4. Use the definition to prove: [5 pts]

$$\lim_{x \to 3} (1 - x) = -2$$

You do not need to prove any sequence convergence.

Solution:

5. Use the definition to prove: [5 pts]

$$\lim_{x \to 10} 5x + 2 \neq 42$$

You do not need to prove any sequence convergence.

Solution:

6. Prove that the following function is not continuous at $x = 2$: [5 pts]

$$f(x) = \begin{cases} 4x & \text{if } x < 2 \\ 3x^2 & \text{if } x \geq 2 \end{cases}$$

You do not need to prove any sequence convergence.

Solution:

7. Prove that $f(x) = 3 - 4x$ is continuous at $x = 10$. You do not need to prove any sequence convergence. [5 pts]

Solution:

8. Prove that the set of infinite binary strings like $b_1b_2b_3...$ with $b_i \in \{0, 1\}$ is uncountable. [11 pts]

Solution:

9. Suppose $S = [10, 15)$. Prove that the supremum is 15. [12 pts]

Solution:

10. Prove using the definition of closed that $[2, \infty)$ is closed. [12 pts]

Solution:

11. Prove that a sequence of numbers greater than or equal to 10 which converges must converge to a number greater than or equal to 10. [15 pts]

Solution: