MATH310 Exam 3 Sample Questions

1. Write down the definition of $\{a_n\} \longrightarrow a_0$ and then write down its negation with the negation pushed through then simplified.

2. Use the definition to prove:
$$\left\{\frac{2n}{1-3n}\right\} \longrightarrow -\frac{2}{3}$$

3. Use the definition to prove:
$$\left\{\frac{5^n-3}{6^n+1}\right\} \longrightarrow 0$$

4. Use the definition to prove:
$$\{(-1)^n\} \not\longrightarrow 0$$

5. Use the definition to prove:
$$\{\sqrt{n}\} \not\longrightarrow 5$$

6. Use the definition to prove that the following sequence does not converge:
$$\{\sqrt{n}\}$$

7. Prove that a sequence of numbers greater than or equal to 1 which converges must converge to a number greater than or equal to 1.

8. Write down the definition of $\lim_{x \to x_0} f(x) = L$ and then write down its negation with the negation pushed through then simplified.

9. Use the definition to prove:
$$\lim_{x \to 3} 1 - x = -2$$

10. Use the definition to prove:
$$\lim_{x \to -1} \frac{x+1}{x+3} = 0$$

11. Use the definition to prove:
$$\lim_{x \to 10} 5x + 2 \neq 42$$

12. Write down the definition of "$f(x)$ is continuous at $x = x_0$" and then write down its negation with the negation pushed through then simplified.

13. Prove that the following function is not continuous at $x = 2$:
$$f(x) = \begin{cases} 4x & \text{if } x \leq 2 \\ 3x^2 & \text{if } x > 2 \end{cases}$$

14. Prove that $f(x) = 3 - 4x$ is continuous at $x = 10$.

15. Prove that the following set is countable:
$$\left\{(a,b) \,\middle|\, a, b \in \mathbb{Z} \wedge a < b\right\}$$

16. Prove that the set of infinite binary strings like 1010111... is uncountable.
17. Prove that:
$$|(-3,\infty)| = |\mathbb{R}|$$
18. Prove that:
$$|\mathbb{Z}| = \left|\left\{(a,b,c)\,\middle|\, a,b,c \in \mathbb{Z}\right\}\right|$$
19. Prove that if A is a set with a supremum then the supremum must be unique.
20. Prove that if A is a set with a maximum then the maximum must be unique.
21. Prove using the definition of open that $(1,3)$ is open.
22. Prove using the definition of open that $[1,3)$ is not open.
23. Prove using the definition of open that $(1,\infty)$ is open.
24. Prove using the definition of closed that $(1,3)$ is not closed.
25. Prove using the definition of closed that $[1,3]$ is closed.
26. Prove using the definition of closed that $[1,\infty)$ is closed.
27. (Challenging 410 problem) Prove using the definition of closed that $S = \{0,5\}$ is closed.
28. Prove using the definition of closed that $\mathbb{R}$ is closed.
29. Prove using the definition of open that $\mathbb{R}$ is open.
30. (Challenging 410 problem) Suppose $f : \mathbb{R} \to \mathbb{R}$ has the property that f is continuous and $f(x) = 0$ for all $x \in S$, where $S \subseteq \mathbb{R}$ is dense. Prove that $f(x) = 0$ for all x.