MATH310 Homework 2022-07-14
Due Gradescope 11:59pm 2022-07-18

1. Determine with brief justification if the following are true or false.

   (a) $\forall n \in \mathbb{Z}$, $\frac{1}{3}(n-2) \in \mathbb{Z}$. [3 pts]

   (b) $\exists n \in \mathbb{Z}$, $\frac{1}{3}(n-2) \in \mathbb{Z}$. [3 pts]

   (c) $\exists! n \in \mathbb{Z}$, $\frac{1}{3}(n-2) \in \mathbb{Z}$. [3 pts]

   (d) $\exists! n \in \{0,1,2,3,4\}$, $\frac{1}{3}(n-2) \in \mathbb{Z}$. [3 pts]

   (e) $\forall x \in \mathbb{R}$, $x^2+3 \geq 0$. [3 pts]

   (f) $\exists x \in \mathbb{R}$, $x^2+3 \geq 0$. [3 pts]

   (g) $\forall x \in \{1,2,3\}$, $3x+1$ is prime. [3 pts]

   (h) $\exists x \in \{1,2,3\}$, $3x+1$ is prime. [3 pts]

   (i) $\exists! x \in \{1,2,3\}$, $3x+1$ is prime. [3 pts]

   (j) $\exists x, y \in \mathbb{Z}, x^2 - y^2 = 9$. [3 pts]

2. Of the following only one is true. Identify which is true and which is false, and justify. [10 pts]

$$\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}, y^2 = x$$

$$\exists y \in \mathbb{R}, \forall x \in \mathbb{R}^+, y^2 = x$$

3. Distribute the negation signs for each of the following, adjusting other symbols accordingly.

   (a) $\sim (\forall x, (\sim P(x))) \equiv ?$ [5 pts]

   (b) $\sim (\exists x, \sim P(x) \wedge Q(x)) \equiv ?$ [5 pts]

   (c) $\sim (\exists! x, P(x)) \equiv ?$ [5 pts]

   (d) $\sim (\forall x, \exists y, P(x,y) \wedge \sim Q(x,y)) \equiv ?$ [5 pts]

   (e) $\sim (\exists x, \forall y, P(x,y) \wedge Q(x,y)) \equiv ?$ [5 pts]

   (f) $\sim (\exists x, \exists y, P(x,y) \leftrightarrow Q(x,y)) \equiv ?$ [5 pts]

4. Negate the following.

   (a) There was once a year in which every day was rainy or snowy. [5 pts]

   (b) For every week there is at least one day where if it's cloudy then it snows. [5 pts]

5. Assume a_n is a sequence of real numbers. The formal definition that a_n converges to $a_0 \in \mathbb{R}$ as $n \to \infty$ is: [10 pts]

$$\forall \epsilon > 0, \exists N \in \mathbb{Z}^+, (n \geq N \to |a_n - a_0| < \epsilon)$$

   Negate this statement.

6. If $P(x)$ is some unknown open sentence find a sentence equivalent to $\exists! x \in \mathbb{R}, P(x)$ which doesn't use ! in it. [10 pts]