1. Determine with brief justification if the following are true or false.

(a) \( \forall n \in \mathbb{Z}, \frac{1}{3}(n - 2) \in \mathbb{Z} \) \[3 \text{ pts}\]
(b) \( \exists n \in \mathbb{Z}, \frac{1}{3}(n - 2) \in \mathbb{Z} \) \[3 \text{ pts}\]
(c) \( \exists n \in \mathbb{Z}, \frac{1}{3}(n - 2) \in \mathbb{Z} \) \[3 \text{ pts}\]
(d) \( \exists n \in \{0, 1, 2, 3, 4\}, \frac{1}{3}(n - 2) \in \mathbb{Z} \) \[3 \text{ pts}\]
(e) \( \forall x \in \mathbb{R}, x^2 + 3 \geq 0 \) \[3 \text{ pts}\]
(f) \( \exists x \in \mathbb{R}, x^2 + 3 \geq 0 \) \[3 \text{ pts}\]
(g) \( \forall x \in \{1, 2, 3\}, 3x + 1 \text{ is prime.} \) \[3 \text{ pts}\]
(h) \( \exists x \in \{1, 2, 3\}, 3x + 1 \text{ is prime.} \) \[3 \text{ pts}\]
(i) \( \exists x \in \{1, 2, 3\}, 3x + 1 \text{ is prime.} \) \[3 \text{ pts}\]
(j) \( \exists x, y \in \mathbb{Z}, x^2 - y^2 = 9 \) \[3 \text{ pts}\]

2. Of the following only one is true. Identify which is true and which is false, and justify. \[10 \text{ pts}\]

\[ \forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}, y^2 = x \]
\[ \exists y \in \mathbb{R}, \forall x \in \mathbb{R}^+, y^2 = x \]

3. Distribute the negation signs for each of the following, adjusting other symbols accordingly. \[5 \text{ pts}\]

(a) \( \sim (\forall x, (\sim P(x))) \equiv ? \)
(b) \( \sim (\exists x, \sim P(x) \land Q(x)) \equiv ? \)
(c) \( \sim (\exists! x, P(x)) \equiv ? \)
(d) \( \sim (\forall x, \exists y, P(x, y) \land \sim Q(x, y)) \equiv ? \)
(e) \( \sim (\exists x, \forall y, P(x, y) \land Q(x, y)) \equiv ? \)
(f) \( \sim (\exists x, \exists y, P(x, y) \leftrightarrow Q(x, y)) \equiv ? \)

4. Negate the following. \[5 \text{ pts}\]

(a) \( \text{There was once a year in which every day was rainy or snowy.} \)
(b) \( \text{For every week there is at least one day where if it’s cloudy then it snows.} \)

5. Assume \( a_n \) is a sequence of real numbers. The formal definition that \( a_n \) converges to \( a_0 \in \mathbb{R} \) as \( n \to \infty \) is:

\[ \forall \epsilon > 0, \exists N \in \mathbb{Z}^+, (n \geq N \to |a_n - a_0| < \epsilon) \]

Negate this statement. \[10 \text{ pts}\]

6. If \( P(x) \) is some unknown open sentence find a sentence equivalent to \( \exists! x \in \mathbb{R}, P(x) \) which doesn’t use \( ! \) in it. \[10 \text{ pts}\]