

**MATH310 Homework 2022-07-28**  
**Due Gradescope 11:59pm 2022-08-01**

1. Define  $A = \{a, b, c, d\}$  and  $B = \{x, y, z\}$  and define the relation  $R$  from  $A$  to  $B$  by

$$R = \{(a, x), (a, z), (b, y), (b, z), (c, y)\}$$

- (a) What is the domain of  $R$ ? [5 pts]
- (b) List the elements in  $\{\alpha \in A \mid \alpha R y\}$ . [5 pts]
- (c) Find  $\left| \{(\alpha, \beta) \mid (\alpha = a) \vee (\beta = z)\} \right|$ . [5 pts]
2. Define  $A = \{1, 2, 3, 4, 5, 6\}$ . Suppose you start defining the relation: [15 pts]

$$R = \{(1, 3), (3, 5), (3, 6), (5, 2), \dots\}$$

Add as many elements as necessary to  $R$  (but no more than necessary) to make sure that the relation is reflexive, symmetric and transitive.

3. Define a relation  $R$  on  $\mathbb{Z}$  by  $R = \{(x, y) \mid xy \geq 0\}$ . Prove or disprove that  $R$  is transitive. [20 pts]
4. Define a relation  $R$  on  $\mathbb{Z}$  by: [25 pts]

$$R = \{(a, b) \mid 4 \mid (3a + b)\}$$

Prove that  $R$  is an equivalence relation.

5. Define a relation  $R$  on  $\mathbb{Z}$  by: [25 pt]

$$R = \{(a, b) \mid a^2 + b^2 \text{ is even}\}$$

Prove that  $R$  is an equivalence relation.