MATH310 Homework 2022-08-03
Due Gradescope 11:59pm 2022-08-05

1. Prove that the function $f : (\mathbb{R} - \{3\}) \to (\mathbb{R} - \{1/2\})$ defined by $f(x) = \frac{x}{2x-6}$. is 1-1 and find a formula for its inverse. [20 pts]

2. Prove that the function $f : \mathbb{R} - \{1\} \to \mathbb{R}$ defined by $f(x) = \frac{1}{(x-1)^2}$ is not 1-1 and therefore has no inverse. [20 pts]

3. Give two distinct functions $f_1, f_2 : [0, 1] \to [0, 1]$ which are both bijective. Prove bijectivity and distinctness. [20 pts]

4. Prove that if $f : A \to B$ and $g : B \to C$ are both injective then so is $g \circ f : A \to C$. [20 pts]

5. Prove that $f : \mathbb{N} \to \mathbb{Z}$ given by $f(n) = \frac{1+(-1)^n(2n-1)}{4}$ is a bijection. [20 pts]
   Note: This is quite challenging - good luck!