1. Given the points: \((-2,2), (-1,1), (1,-1), (2,-1)\)
   (a) Place the points in a matrix \(A\) and find the singular value decomposition of \(A\).
   (b) In which direction do the points have the most variance and what is the proportion of the total variance in that direction?
   (c) If we were to assume that the points are supposed to follow a straight line in the direction of maximum variance (with the other direction being noise we can ignore), which \(y\) value would correspond to \(x = 3\) and which \(x\) value would correspond to \(y = -3\)?
   (d) Find the perpendicular projection of the points onto the direction of maximum variance.

2. The file `makesomepoints.m` on the class webpage creates a matrix \(A\) of 100 points in \(\mathbb{R}^5\).
   (a) Find the matrix \(U\) and the singular values from the SVD of \(A\).
   (b) Find the proportion of the total variance in each of the directions indicated by \(U\).
   (c) What is the minimum number of singular values (going from largest to smallest) that need to be preserved in order to preserve at least 99.9% of the total variance in the data?
   (d) If \(\Sigma'\) is the matrix \(\Sigma\) except with the unnecessary (according to part (c)) singular values set to 0, calculate \(A' = U\Sigma'V^T\) and write down the first three columns. In your opinion are these close to the first three columns of \(A\)?

We can find the Singular Value Decomposition of a matrix \(A\) in Matlab with:

```matlab
>> [U,S,V] = svd(A)
```

If we want to zero out a singular value and recalculate the matrix we can do so as follows. For example if we wanted to create \(\Sigma'\) by zeroing out the (4,4)-entry and create \(A'\) we’d do:

```matlab
>> [U,S,V] = svd(A);
>> SP = S;
>> SP(4,4) = 0;
>> AP = U*SP*transpose(V)
```

If we have a matrix \(A\) in Matlab and we only want the first few columns we can do that as follows. For example for the first three columns:

```matlab
>> A(:,[1:3])
```