Math 406 Exam 1 Spring 2020 **Justin Wyss-Gallifent**

Note: I have ordered these in terms of what I think is increasing difficulty. You may have other opinions! Remember that this exam will be curved, I do not expect you to finish all the problems in 50 minutes.

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1. Write down the prime factorization of 10!. [5 pts]

2. Find the least nonnegative residue of 11^{67} mod 13. [10 pts]

3. Find all incongruent solutions mod 40, as least nonnegative residues, to the following linear congruence: [10 pts]
$$12x \equiv 28 \bmod 40$$

4. Use the Euclidean Algorithm to find $\gcd(390, 72)$ and write this as a linear combination of the two. [10 pts]

5. Use the Chinese Remainder Theorem to find the smallest positive solution to the system: [15 pts]
$$\begin{aligned} x &\equiv 2 \bmod 5 \\ x &\equiv 1 \bmod 6 \\ x &\equiv 4 \bmod 7 \end{aligned}$$

6. Use mathematical induction to prove that: [10 pts]
$$n! \geq n^3 \text{ for } n \geq 6$$

7. One of the following two sets is well-ordered and one is not. Decide which is which and justify. You may assume only that $\mathbb{Z}^+$ is well-ordered. [15 pts]
$$\begin{aligned} S_1 &= [0,1] \cap \mathbb{Q} \\ S_2 &= \left\{1 - 2^k \mid k \in \mathbb{Z}^+\right\} \end{aligned}$$

8. Use the Fundamental Theorem of Arithmetic (uniqueness of prime factorization) to prove that $\sqrt{2}$ is irrational. Hint: Use contradiction. [10 pts]

9. Suppose $a, b, c, d \in \mathbb{Z}$ with $a \mid c$, $b \mid c$, $d = \gcd(a, b)$ and $d^2 \mid c$. Prove that $ab \mid c$. [15 pts]