MATH 406 (JWG) Exam 1 Spring 2021

Due by Wed Mar 3 at 12:00pm

Exam Logistics:

1. From the moment you download this exam you have three hours to take the exam and submit to Gradescope. This includes the entire upload and tag procedure so do not wait until the last minute to do these things.
2. Tag your problems! Please! Pretty please!
3. You may print the exam, write on it, scan and upload.
4. Or you may just write on it on a tablet and upload.
5. Or you are welcome to write the answers on separate pieces of paper if other options don't appeal to you, then scan and upload.

Exam Rules:

1. You may ask for clarification on questions but you may not ask for help on questions!
2. You are permitted to use official class resources which means your own written notes, class Panopto recordings and the textbook.
3. You are not permitted to use other resources. Thus no friends, internet, calculators, Wolfram Alpha, etc.
4. By taking this exam you agree that if you are found in violation of these rules that the minimum penalty will be a grade of 0 on this exam.

Exam Work:

1. Show all work as appropriate for and using techniques learned in this course.
2. Any pictures, work and scribbles which are legible and relevant will be considered for partial credit.
3. Arithmetic calculations do not need to be simplified unless specified.

1. Determine if the following set is well-ordered and justify: [5 pts]

$$\left\{ \frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0,\ a > b \right\}$$

Solution:

2. Simplify: [10 pts]

$$\sum_{j=2}^{n} \left(3^j + \frac{j+1}{2} \right)$$

Solution:

3. Use the Prime Number Theorem to find the approximate number of primes between 1000 and 2000. [5 pts]

   Solution:

4. Suppose $a, b \in \mathbb{Z}$ satisfy $\gcd(a, b) = 2 \cdot 5^2$ and $\operatorname{lcm}(a, b) = 2^2 \cdot 5^2 \cdot 7^2$. What could a and b be? Justify. [5 pts]

   Solution:

5. Find all solutions mod $210 = 3 \cdot 5 \cdot 14$ to the following system of congruences: [10 pts]

$$
\begin{aligned}
x &= 2 \bmod 3 \\
x &= 2 \bmod 5 \\
2x &= 6 \bmod 14
\end{aligned}
$$

Solution:

6. Find the least nonnegative residue of each of the following. Justify. [10 pts]

(a) $1! + 2! + 3! + ... + 100!$ modulo 15.
Solution:

(b) 23471^{589227} mod 23472.
Solution:

(c) 3^{260} mod 15.
Solution:

7. Find all (if any) incongruent solutions modulo the given modulus to the following: [10 pts]

(a) $35x \equiv 10 \mod 55$

Solution:

(b) $2x \equiv 13^{162} \mod 20$

Solution:

8. Suppose $n = abc$ is a three-digit number, where each letter is a digit. (For example, 347 has $a = 3$, $b = 4$ and $c = 7$.) Prove that $11 \mid n$ if and only if $11 \mid (a - b + c)$. [10 pts]

Solution:

9. Use mathematical induction to prove: [10 pts]

$$1(1!) + 2(2!) + ... + n(n!) = (n+1)! - 1 \text{ for all integers } n \geq 1$$

Solution:

10. Prove that if $a, b, c \in \mathbb{Z}$ with $\gcd(a, b) = 1$ and $c \big| (a + b)$ then $\gcd(c, a) = \gcd(c, b) = 1$. [10 pts]

Solution:

11. Prove that a perfect square cannot have exactly four distinct positive divisors. [5 pts]

Solution:

12. Prove using the Uniqueness of Prime Factorizations that $\sqrt{6}$ is irrational. [10 pts]

Solution: