Math 406 Exam 1 **Justin Wyss-Gallifent**

Directions: Each numbered question is worth ten points. Coherent, logical justification is required for all problems except those in which counterexamples are requested, those can just be given, or for simple computations.

Note: I've ordered these by difficulty as I perceive it. Your opinion on difficulty might vary, but knowing how I ordered them might help you decide which to do first and which to do last!

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1. (a) Find $\pi(18)$.
   (b) Show that the set $\left\{ \frac{a}{b} \mid a, b \in \mathbb{Z}^+, a > b \right\}$ is not well-ordered.
   (c) Find how many primes there are, approximately, between one billion and two billion.
2. Find all integer solutions to $115x + 25y = 10$.
3. Find the number of zeros at the end of 1000! with justification.
4. The following are all false. Provide explicit numerical counterexamples.
   (a) $a \mid bc$ implies $a \mid b$ or $a \mid c$.
   (b) $a \mid b$ and $a \mid c$ implies $b \mid c$.
   (c) $3 \mid a$ and $3 \mid b$ implies $\gcd(a, b) = 3$.
5. Simplify $\prod_{j=1}^{n} \left(1 + \frac{2}{j}\right)$. Your result should not have a $\prod$ in it, or any sort of long product.
6. Use Mathematical Induction to prove $2^1 + 2^2 + ... + 2^n = 2^{n+1} - 2$ for all integers $n \geq 1$.
7. Find all $n \in \mathbb{Z}$ with $n^2 - 5n + 6$ prime.
8. Suppose p is a prime and a is a positive integer less than p. Find all possibilities for $\gcd(a, 7a+p)$.
9. Use the Fundamental Theorem of Arithmetic to prove that $\sqrt{6}$ is irrational.
10. Prove that for $a, b \in \mathbb{Z}$ and $n \in \mathbb{Z}^+$ that if $a^n \mid b^n$ then $a \mid b$.

The End ⌣̈