1. Show that 91 is a Fermat Pseudoprime to the base 3. Note that 91 is not prime!

2. Prove that if $n \geq 2$ and $\text{gcd}(6, n) = 1$ then $\phi(3n) = 2\phi(2n)$.

3. Classify all numbers $n$ for which $\tau(n) = 12$.

4. Suppose $n$ is a perfect number and $p$ is a prime such that $pn$ is also perfect. Prove $\text{gcd}(p, n) \neq 1$.

5. Prove that $a^{\phi(b)} + b^{\phi(a)} \equiv 1 \mod ab$ if $\text{gcd}(a, b) = 1$.

6. Suppose that $p$ is prime and $n \in \mathbb{Z}^+$. Prove that $p \not| n$ iff $\phi(pm) = (p - 1)\phi(n)$.

7. (a) Show that 3 is a primitive root modulo 17.
   (b) Find all primitive roots modulo 17.

8. A partial table of indices for 7, a primitive root of 13 is given here:

<table>
<thead>
<tr>
<th>$a$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>ind$_7a$</td>
<td>12</td>
<td>$b$</td>
<td>8</td>
<td>10</td>
<td>3</td>
<td>7</td>
<td>$a$</td>
<td>9</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

   (a) Find $a$ and $b$.
   (b) Use the table to solve the congruence $3^{x-1} \equiv 5 \mod 13$.
   (c) Use the table to solve the congruence $4x^5 \equiv 11 \mod 13$.

9. Suppose $\text{ord}_p a = 3$, where $p$ is an odd prime. Show $\text{ord}_p(a + 1) = 6$.

10. Suppose $r$ is a primitive root modulo $m$, and $k$ is a positive integer with $\text{gcd}(k, \phi(m)) = 1$. Prove $r^k$ is also a primitive root.
    Note: The intention is to do this without the theorem from class.