MATH 406 (JWG) Exam 2 Spring 2021 Sample 1

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1. Show that 91 is a Fermat Pseudoprime to the base 3. Note that 91 is not prime!
2. Prove that if $n \geq 2$ and $\gcd(6, n) = 1$ then $\phi(3n) = 2\phi(2n)$.
3. Classify all numbers n for which $\tau(n) = 12$.
4. Suppose n is a perfect number and p is a prime such that pn is also perfect. Prove $\gcd(p, n) \neq 1$.
5. Prove that $a^{\phi(b)} + b^{\phi(a)} \equiv 1 \mod ab$ if $\gcd(a, b) = 1$.
6. Suppose that p is prime and $n \in \mathbb{Z}^+$. Prove that $p \nmid n$ iff $\phi(pn) = (p-1)\phi(n)$.
7. (a) Show that 3 is a primitive root modulo 17.

   (b) Find all primitive roots modulo 17.
8. A partial table of indices for 7, a primitive root of 13 is given here:

a	1	2	3	4	5	6	7	8	9	10	11	12
$\text{ind}_7 a$	12	b	8	10	3	7	a	9	4	2	5	6

   (a) Find a and b.

   (b) Use the table to solve the congruence $3^{x-1} \equiv 5 \mod 13$.

   (c) Use the table to solve the congruence $4x^5 \equiv 11 \mod 13$.
9. Suppose $\text{ord}_p a = 3$, where p is an odd prime. Show $\text{ord}_p(a+1) = 6$.
10. Suppose r is a primitive root modulo m, and k is a positive integer with $\gcd(k, \phi(m)) = 1$. Prove r^k is also a primitive root.

    Note: The intention is to do this without the theorem from class.