MATH 406 (JWG) Exam 2 Spring 2021 Sample 2

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1. Calculate:
   (a) $\phi(2^3 \cdot 5 \cdot 11^2)$
   (b) $\sigma(200)$
   (c) $\tau(2000)$
2. Use Wilson's Theorem to find the remainder when 16! is divided by 19.
3. Find all n with $\phi(n) = 16$.
4. Show that 25 is a Fermat Pseudoprime to the base 7.
5. An abundant number is a number n with $\sigma(n) > 2n$. Prove that there are infinitely many even abundant numbers by finding one abundant number and by showing that if n is abundant and a prime p satisfies $p \nmid n$ then pn is also abundant.
6. A partial table of indices for 2, a primitive root of 13, is given here:

a	1	2	3	4	5	6	7	8	9	10	11	12
$\text{ind}_2 a$	12	1	4	2	9	5	11	3	a	b	7	6

   (a) Find a and b with justification.
   (b) Use the table to solve the congruence $3^{2x+1} \equiv 9 \mod 13$.
   (c) Use the table to solve the congruence $7x^5 \equiv 3 \mod 13$.
7. Prove that if $\text{ord}_n a = hk$ then $\text{ord}_n \left(a^h\right) = k$.
   Note: The intention is to do this without the theorem from class.
8. Let r be a primitive root for an odd prime p. Prove that $\text{ind}_r(p-1) = \frac{1}{2}(p-1)$.
9. Find all positive integers n such that $\phi(n)$ is prime. Explain!
10. Show that if a is relatively prime to m and $\text{ord}_m a = m - 1$ then m is prime.