1. Calculate:
   (a) $\phi(2^3 \cdot 5 \cdot 11^2)$
   (b) $\sigma(200)$
   (c) $\tau(2000)$

2. Use Wilson’s Theorem to find the remainder when $16!$ is divided by $19$.

3. Find all $n$ with $\phi(n) = 16$.

4. Show that 25 is a Fermat Pseudoprime to the base 7.

5. An abundant number is a number $n$ with $\sigma(n) > 2n$. Prove that there are infinitely many even abundant numbers by finding one abundant number and by showing that if $n$ is abundant and a prime $p$ satisfies $p \nmid n$ then $pn$ is also abundant.

6. A partial table of indices for 2, a primitive root of 13, is given here:

<table>
<thead>
<tr>
<th>$a$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>ind$_2^a$</td>
<td>12</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>9</td>
<td>5</td>
<td>11</td>
<td>3</td>
<td>$a$</td>
<td>$b$</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

   (a) Find $a$ and $b$ with justification.
   (b) Use the table to solve the congruence $3^{2^x+1} \equiv 9 \mod 13$.
   (c) Use the table to solve the congruence $7x^5 \equiv 3 \mod 13$.

7. Prove that if $\text{ord}_n a = hk$ then $\text{ord}_n (a^h) = k$.
   
   Note: The intention is to do this without the theorem from class.

8. Let $r$ be a primitive root for an odd prime $p$. Prove that $\text{ind}_r (p-1) = \frac{1}{2}(p-1)$.

9. Find all positive integers $n$ such that $\phi(n)$ is prime. Explain!

10. Show that if $a$ is relatively prime to $m$ and $\text{ord}_m a = m - 1$ then $m$ is prime.