1. (a) Show that 5 is a primitive root mod 14.
   (b) Construct a table of indices mod 14 using the primitive root 5.
   (c) Use this table to solve the congruence \( x^3 \equiv 13 \mod 14 \).
   (d) Use this table to solve the congruence \( 11^{1-x} \equiv 9 \mod 14 \).

For problems 2-5:
Let \( m = 29 \) and let \( r = 8 \). It is a fact that \( r = 8 \) is a primitive root modulo 29 (don’t prove this). Consider the following table of indices. Note that three are missing.

<table>
<thead>
<tr>
<th>( a )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{ind}_r a )</td>
<td>28</td>
<td>19</td>
<td>11</td>
<td>10</td>
<td>26</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>a</td>
<td>17</td>
<td>27</td>
<td>21</td>
<td>6</td>
<td>b</td>
</tr>
<tr>
<td>( a )</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
</tr>
<tr>
<td>( \text{ind}_r a )</td>
<td>9</td>
<td>20</td>
<td>7</td>
<td>13</td>
<td>3</td>
<td>8</td>
<td>15</td>
<td>18</td>
<td>16</td>
<td>12</td>
<td>c</td>
<td>25</td>
<td>5</td>
<td>14</td>
</tr>
</tbody>
</table>

2. Solve for \( a, b \) and \( c \). There are several ways to do this. Do it however you choose, just be clear about your work.

3. Using the table of indices solve \( 19x^4 \equiv 21 \mod 29 \).

4. Using the table of indices, solve \( 27x^3 \equiv 24 \mod 29 \).

5. Using the table of indices, solve \( 3^{2x} \equiv 23 \mod 29 \).

6. With logarithms we have \( \log_r (a/b) = \log_r a - \log_r b \).
   (a) Why is it not reasonable to write \( \text{ind}_r \left( \frac{a}{b} \right) \equiv \text{ind}_r a - \text{ind}_r b \mod \phi(n) \) when \( a, b \) are coprime to \( n \) and \( r \) is a primitive root?
   (b) What would be a reasonable index substitute for this logarithm rule?
   (c) Prove this substitute.

7. Suppose \( p \) is an odd prime and both \( r_1 \) and \( r_2 \) are primitive roots for \( p \). Prove that \( r_1 r_2 \) is not a primitive root for \( p \).

8. Determine, by squaring, which of 1, ..., 16 are quadratic residues of \( p = 17 \).

9. Calculate \( \left( \frac{3}{17} \right) \) by
   (a) Euler’s Criterion
   (b) Gauss’s Lemma

10. Calculate each of the following using properties of the Legendre Symbol, not by raw calculation.
    (a) \( \left( \frac{2}{17} \right) \)
    (b) \( \left( \frac{-1}{17} \right) \)
    (c) \( \left( \frac{8}{17} \right) \)

11. Prove that if \( p \) and \( q = 2p + 1 \) are both odd primes then \(-4\) is a primitive root of \( q \).

12. Prove that if \( p \equiv 1 \mod 4 \) is a prime then \(-4\) and \((p - 1)/4\) are both quadratic residues of \( p \).

13. Show that \(-1\) is a quadratic residue for only one out of every pair of twin primes.