**Math 406 Section 1.2: Sums and Products**

1. **Sums:**

   (a) **Definition:** For a sequence $a_1, a_2, ...$ we know the notation

   $$\sum_{i=1}^{n} a_i = a_1 + a_2 ... + a_n$$

   so there's not much that needs to be said.

   (b) **Geometric Sum:** It is useful to remember the geometric sum formula:

   $$\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r - 1}$$

   which follows from the polynomial product:

   $$(1 + r + r^2 + ... + r^n)(r - 1) = r^{n+1} - 1$$

   (c) **Telescoping Sums:** It is also useful to recall that some sums telescope closed, for example:

   $$\begin{aligned} \sum_{i=1}^{n} \frac{1}{j(j+1)} &= \sum_{i=1}^{n} \left( \frac{1}{j} - \frac{1}{j+1} \right) \\ &= \left( \frac{1}{1} - \frac{1}{2} \right) + \left( \frac{1}{2} - \frac{1}{3} \right) + \left( \frac{1}{3} - \frac{1}{4} \right) + ... + \left( \frac{1}{n} - \frac{1}{n+1} \right) \\ &= \frac{1}{1} - \frac{1}{n+1} \end{aligned}$$

2. **Products:** Similarly we define the product:

   $$\prod_{i=1}^{n} a_i = a_1 a_2 ... a_n$$