1. Given $A = 3827382$ and $B = 61329086$. [15 pts]
   (a) Find the prime factorizations of $A$ and $B$ and use them to find $\gcd(A, B)$. 
   (b) Find $\gcd(A, B)$ using the Euclidean Algorithm.

2. Use the Chinese Remainder Theorem to find the smallest and second smallest nonnegative solutions to the system:
   \[ \begin{align*}
   x &\equiv 11 \pmod{26} \\
   x &\equiv 26 \pmod{27} \\
   x &\equiv 15 \pmod{17}
   \end{align*} \]

3. For each of $n = 85, 430, 8833, 54861$ find the exact value $p_n$ of the $n^{th}$ prime (however you want) and then approximate value $a_n$ of the $n^{th}$ prime (using the Prime Number Theorem Corollary). Calculate the percentage error
   \[ \frac{100|p_n - a_n|}{p_n} \]
   for each.

4. Find all incongruent solutions mod 120 to the linear system: [10 pts]
   \[ 32x \equiv 112 \pmod{120} \]

5. Find all primitive roots for $n = 29$ as follows: First find the smallest positive primitive root. Then use the Theorem from class which yields all the remaining ones. Final answers should be least nonnegative residues.

6. It’s a fact that $r = 7$ is a primitive root mod 13. [15 pts]
   (a) Use this to construct a table of indices for this primitive root.
   (b) Use the table of indices to solve the equation: $x^2 \equiv 12 \pmod{13}$. Your answer(s) should be mod 13.
   (c) Use the table of indices to solve the equation: $4^x \equiv 12 \pmod{13}$. Your answer(s) should be mod 12.

7. Calculate the following Jacobi symbols: [15 pts]
   (a) $\left( \frac{1613}{47} \right)$
   (b) $\left( \frac{6}{39997} \right)$
8. Suppose you intercept the following ciphertext from Alice to Bob: 

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9393 14933 17090 16839 5198 7076 17527 11323 6142 11303
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You know that Bob’s public key is \((e, n) = (1367, 27331)\). Bob thinks this is secure because he doesn’t believe that his \(n\) can be factored easily. Factor \(n = 27331\), find \(\phi(n)\), find \(d\) and then decrypt the message. Be clear about the steps you take.

9. Determine if each of the following sets is well-ordered. If a set is not well-ordered give evidence. If a set is well-ordered no evidence is required. 

(a) \(3\mathbb{Z}\)  
(b) \(\{0\} \cup \left\{ \frac{n+5}{n} \mid n \in \mathbb{Z}^+ \right\}\)  
(c) \(\left\{ \lfloor \sqrt{n} \rfloor \mid n \in \mathbb{Z}^+ \right\}\)

10. Suppose \(p \geq 47\) is an unknown prime. Find all solutions to \(x^2 + 45 \equiv 14x \mod p\). Note that your solutions will be mod \(p\).

11. Consider the inequality: 

\(8^n < n!\)

(a) Find the smallest positive integer \(n_0\) for which this is true. Do this however you wish.  
(b) Prove by induction that \(8^n < n!\) for all \(n \geq n_0\).

12. Suppose \(p\) is an odd prime such that there is some \(a\) so that \(a\) is a quadratic residue of \(p\) but \(2a\) is a quadratic non-residue of \(p\). Prove that \(p \equiv \pm 3 \mod 8\).

13. Prove that for \(a, b \in \mathbb{Z}\) and \(n \in \mathbb{Z}^+\) that if \(a^n \mid b^n\) then \(a \mid b\).

14. Prove that if \(a, b, c \in \mathbb{Z}\) with \(\gcd(a, b) = 1\) and \(c \mid (a + b)\) then \(\gcd(c, a) = \gcd(c, b) = 1\).